Week 9 Wednesday

March 4, 2020 8:59 AM

4.2 Determinants Cont'd

Ex: Solve using Cramer's Rule

$$\begin{cases} 2x + 3y + 2z = -11 \\ 3x \qquad + 5z = 23 \\ 4x + y + z = 1 \end{cases}$$

$$|A| = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 0 & 5 \\ 4 & 1 & 1 \end{vmatrix} = -3(1) - 5(-10) = 47 \qquad \begin{bmatrix} + & - & \\ - & + & \\ & & \ddots \end{bmatrix}$$

(2nd row)

$$A_1 = \begin{bmatrix} -11 & 3 & 2 \\ 23 & 0 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

$\vec{b}$

$$|A_1| = \begin{vmatrix} -11 & 3 & 2 \\ 23 & 0 & 5 \\ 1 & 1 & 1 \end{vmatrix} = -23(1) - 5(-14) = 47$$

$$|A_2| = \begin{vmatrix} 2 & -11 & 2 \\ 3 & 23 & 5 \\ 4 & 1 & 1 \end{vmatrix} = 2(18) + 11(-17) + 2(-89) = -329$$

$\vec{b}$

$$|A_3| = \begin{vmatrix} 2 & 3 & -11 \\ 3 & 0 & 23 \\ 4 & 1 & 1 \end{vmatrix} = 188$$

$\vec{b}$

$$x = \frac{|A_1|}{|A|} = \frac{47}{47} = 1$$

$$y = \frac{|A_2|}{|A|} = \frac{-329}{47} = -7$$

$$z = \frac{|A_3|}{|A|} = \frac{188}{47} = 4$$

Def

Cofactor: The signed determinant associated with a matrix entry

Ex: $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 5 \end{bmatrix}$

$\begin{bmatrix} + & - & \\ - & + & \\ & & \cdots \end{bmatrix}$

Cofactor $C_{11} = +\begin{vmatrix} 0 & 3 \\ 1 & 5 \end{vmatrix} = -3$

$C_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = -2$

$C_{23} = -\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = -3$

Cofactor Matrix $= \begin{bmatrix} -3 & -2 & 1 \\ 11 & 4 & -3 \\ -6 & -2 & 2 \end{bmatrix}$

Def

The adjoint of A: transpose of the cofactor matrix

Written adj(A)

FACT

If A is $n \times n$ and $|A| \neq 0$ then

$$A^{-1} = \frac{1}{|A|}\,\mathrm{adj}(A)$$

Ex: $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 5 \end{bmatrix}$

Find A^{-1} using the adjoint

$|A| = -1(-11) - 3(3) = 2$ (2nd row) $\begin{bmatrix} + \\ - & + & - \\ \end{bmatrix}$

Cofactor Matrix $= \begin{bmatrix} -3 & -2 & 1 \\ 11 & 4 & -3 \\ -6 & -2 & 2 \end{bmatrix}$

transpose

$\mathrm{adj}(A) = \begin{bmatrix} -3 & 11 & -6 \\ -2 & 4 & -2 \\ 1 & -3 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{2}\,\mathrm{adj}(A)$$

Ex: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Find A^{-1} using the adjoint

$|A| = ad - bc$

$C_{11} = +d$

$C_{12} = -\det(c) = -c$

$\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

$C_{21} = -\det(b) = -b$

$C_{22} = a$

Cofactor Matrix $= \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

transpose

$\mathrm{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Review

1. $A = \begin{bmatrix} 1 & -1 & 7 \\ 2 & 3 & 4 \\ 5 & 0 & 25 \end{bmatrix}$

Find a basis for:

a) Col(A)

b) null(A)

$$\begin{array}{c} R_2 - 2R_1 \\ R_3 - 5R_1 \end{array} \begin{bmatrix} 1 & -1 & 7 \\ 0 & 5 & -10 \\ 0 & 5 & -10 \end{bmatrix}$$

$$\frac{R_2}{5} \begin{bmatrix} 1 & -1 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -10 \end{bmatrix}$$

$$\begin{array}{c} R_1 + R_2 \\ \\ R_3 - 5R_2 \end{array} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{RREF}$$

a) $\begin{bmatrix} 1 & & \\ & 1 & \end{bmatrix}$

Use Columns 1 and 2 of $\underline{\underline{A}}$

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \right\}$$

b) $\text{null}(A) = \{ \vec{x} \text{ that solve } A\vec{x} = \vec{0} \}$

$$[A \mid \vec{0}]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\boxed{x_3 = t}$$

$$x_1 + 5x_3 = 0 \longrightarrow \boxed{x_1 = -5t}$$

$$\boxed{x_2 = 2t}$$

$$\vec{x} = \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix} t$$

$$\text{Basis} = \left\{ \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix} \right\}$$