Week 9 Monday

March 2, 2020 7:57 AM

Test FRI MARCH 6

2.3-2.4, 3.1-3.5

Practice Problems, Videos } www.leahhoward.com

Bring Earplugs / Music

4.1 Eigenvalues and Eigenvectors

Ex: Find a basis for the eigenspace E_8 for $A = \begin{bmatrix} 9 & -2 \\ 5 & -2 \end{bmatrix}$

Solve $[A - \lambda I \mid \vec{0}]$

$[A - 8I \mid \vec{0}]$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 5 & -10 & 0 \end{array}\right]$$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \text{RREF}$$

$x_2 = t$

$x_1 - 2x_2 = 0 \rightarrow x_1 = 2t$

eigenvectors $\vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} t \quad (t \neq 0)$

basis $= \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$

FACT

$$\boxed{\begin{array}{l}\text{Let } B \text{ be an } n \times n \text{ matrix.} \\ B\vec{x} = \vec{0} \text{ has nontrivial solutions} \\ \text{exactly when } \det B = 0\end{array}}$$

FACT

$\boxed{\text{To find eigenvalues of } A, \text{ solve } \det(A-\lambda I)=0}$

Why? λ is an eigenvalue of A

if and only if $A\vec{x} = \lambda \vec{x}$ $(\vec{x} \neq \vec{0})$

if and only if $(A-\lambda I)\vec{x} = \vec{0}$ $(\vec{x} \neq \vec{0})$

if and only if $\det(A-\lambda I) = 0$

Ex: Find all eigenvalues of $A = \begin{bmatrix} 4 & -2 \\ 5 & -7 \end{bmatrix}$

Solve $\det(A-\lambda I) = 0$

$$\begin{vmatrix} 4-\lambda & -2 \\ 5 & -7-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)(-7-\lambda) - (-2)(5) = 0$$

$$-28 - 4\lambda + 7\lambda + \lambda^2 + 10 = 0$$

$$\lambda^2 + 3\lambda - 18 = 0$$

$$(\lambda+6)(\lambda-3) = 0$$

$$\lambda = -6, 3$$

Summary

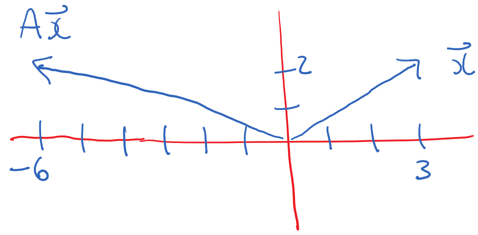
To find eigenvectors, solve $[A-\lambda I \mid \vec{0}]$ (system)

eigenvalues, solve $\det(A-\lambda I) = 0$ (equation)

Ex: $A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$

Find the eigenvalues and eigenvectors geometrically.

$$A\vec{x} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2x_1 \\ x_2 \end{bmatrix}$$

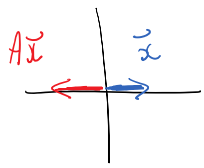


x-axis

$$A\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-2 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}-2\\0\end{bmatrix}$$

$\lambda = -2$

eigenvector $\vec{x} = \begin{bmatrix}1\\0\end{bmatrix}$

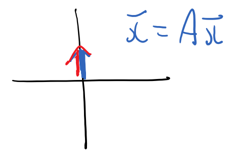


y-axis

$$A\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}-2 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$$

$\lambda = 1$

eigenvector $\vec{x} = \begin{bmatrix}0\\1\end{bmatrix}$



$$\boxed{\begin{aligned} E_{-2} &= \text{span}\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \\ E_1 &= \text{span}\left(\begin{bmatrix}0\\1\end{bmatrix}\right)\end{aligned}}$$

4.2 Determinants

(Return to eigenvalues in 4.3)

- Can write $\det A$ or $|A|$
- Determinants only defined for square matrices

Cofactor Expansion

- Can expand along any row/column
- Checkerboard pattern for signs $\begin{bmatrix} + & - & + \\ - & + & \ddots \end{bmatrix}$

Ex: $A = \begin{bmatrix}4 & 1 & 6\\1 & 2 & 3\\6 & 0 & 7\end{bmatrix}$

Find $\det A$

2^{nd} Column :

$$\begin{vmatrix} 4 & 1 & 6 \\ 1 & 2 & 3 \\ 6 & 0 & 7 \end{vmatrix} = -1\begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} + 2\begin{vmatrix} 4 & 6 \\ 6 & 7 \end{vmatrix} - 0\begin{vmatrix} 4 & 6 \\ 1 & 3 \end{vmatrix}$$

$$= -1(-11) + 2(-8) + 0$$

$$= -5$$

Alternatively :

3^{rd} Row :

$$\begin{vmatrix} 4 & 1 & 6 \\ 1 & 2 & 3 \\ 6 & 0 & 7 \end{vmatrix} = 6\begin{vmatrix} 1 & 6 \\ 2 & 3 \end{vmatrix} - 0 \cancel{\quad} + 7\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 6(-9) + 7(7)$$

$$= -5$$

$$\begin{bmatrix} + & & \\ - & & \\ + & & \end{bmatrix}$$

Ex :

$$\begin{vmatrix} 1 & 6 & 2 & 3 \\ 0 & 0 & 0 & 4 \\ 2 & 1 & 1 & 6 \\ 2 & 0 & 5 & 7 \end{vmatrix} = 4\begin{vmatrix} 1 & 6 & 2 \\ 2 & 1 & 1 \\ 2 & 0 & 5 \end{vmatrix}$$

$$\begin{bmatrix} + & & & \\ - & + & - & + \\ & & & \end{bmatrix}$$

$$= 4\left[2\begin{vmatrix} 6 & 2 \\ 1 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix}\right]$$

$$\begin{bmatrix} + & \\ - & \\ + & \end{bmatrix}$$

$$= 4\left[2(4) + 5(-11)\right]$$

$$= -188$$