Week 8 Wednesday

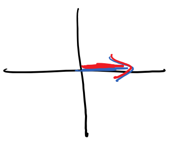
February 26, 2020 8:54 AM

3.6 Linear Transformations Cont'd

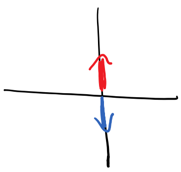
Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$

Reflects a vector in x-axis.

Find $[T]$



$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$



$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\-1\end{bmatrix}$

$$[T] = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

$\uparrow$ $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ $\quad$ $\uparrow$ $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$

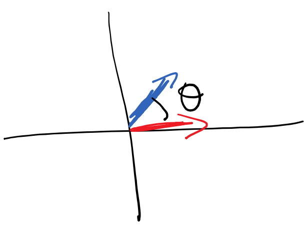
$T: \mathbb{R}^3 \to \mathbb{R}^3$

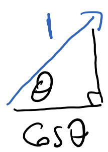
$$[T] = \begin{bmatrix}(\,) & (\,) & (\,)\end{bmatrix}$$

$\uparrow$ $T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)$

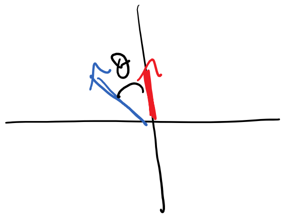
Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$ rotates a vector by angle θ (counterclockwise)

Find $[T]$





$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$



$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix}$$

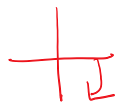
signs

$$[T] = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

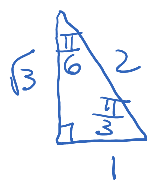
$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ $\quad$ $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$

☆ Know this

Ex: Rotate $\begin{bmatrix}1\\1\end{bmatrix}$ by 30° clockwise



$$[T] = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} \quad \theta = -30^\circ$$



$$\cos(-30^\circ) = \frac{\sqrt{3}}{2}$$
$$\sin(-30^\circ) = -\frac{1}{2}$$

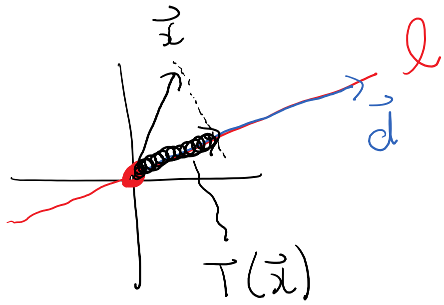
$$[T] = \frac{1}{2}\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{3}\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}\sqrt{3} & 1\\ -1 & \sqrt{3}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}\sqrt{3}+1\\ -1+\sqrt{3}\end{bmatrix}$$

Ex: $T: \mathbb{R}^2 \to \mathbb{R}^2$ projects a vector on the line through the origin with $\vec{d} = \begin{bmatrix} a \\ b \end{bmatrix}$

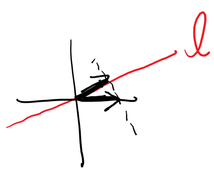
Find $[T]$



$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \text{proj}_{\vec{d}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\vec{d} = \begin{bmatrix} a \\ b \end{bmatrix}$

$$= \frac{\vec{d} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\|\vec{d}\|^2} \vec{d}$$

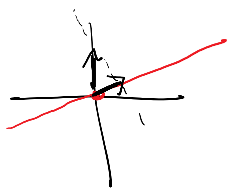


$$= \frac{a}{a^2+b^2} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$= \frac{1}{a^2+b^2} \begin{bmatrix} a^2 \\ ab \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \text{proj}_{\vec{d}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\vec{d} = \begin{bmatrix} a \\ b \end{bmatrix}$



$$= \frac{\vec{d} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\|\vec{d}\|^2} \vec{d}$$

$$= \frac{b}{a^2+b^2} \begin{bmatrix} a \\ b \end{bmatrix}$$

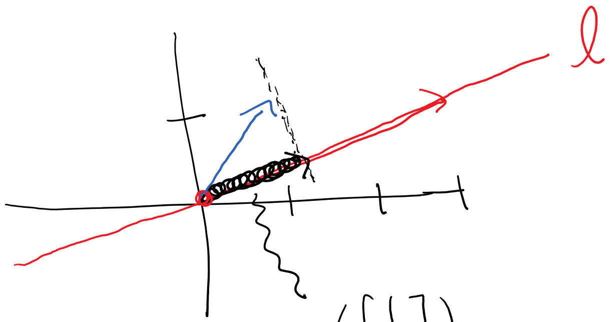
$$= \frac{1}{a^2+b^2} \begin{bmatrix} ab \\ b^2 \end{bmatrix}$$

$$[T] = \frac{1}{a^2+b^2}\begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix}$$

$$\begin{array}{cc} \uparrow & \uparrow \\ T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) & T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) \end{array}$$

★ Know this

Ex: Project $\begin{bmatrix}1\\1\end{bmatrix}$ onto the line through origin with $\vec{d} = \begin{bmatrix}3\\1\end{bmatrix}$.



$$[T] = \frac{1}{a^2+b^2}\begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix} \quad \begin{array}{l} a=3 \\ b=1 \end{array}$$

$$= \frac{1}{10}\begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \frac{1}{10}\begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix}12\\4\end{bmatrix} \text{ or } \begin{bmatrix}6/5\\2/5\end{bmatrix}$$

Apply T_1 then T_2 to $\vec{x}$

Notation : $T_2(T_1(\vec{x}))$ or $(T_2 \circ T_1)(\vec{x})$

Calculation : $[T_2][T_1]\vec{x}$