Week 8 Monday

February 24, 2020 7:57 AM

Test 2 FRI MARCH 6
2.3-2.4, 3.1-3.5 (6 Questions)
Practice Problems on Website

Final Exam Thurs April 16^{th}
1:30pm CC124

3.5 Subspaces and Dimension Cont'd

Recap of row(A), col(A), null(A):
Week 6 Friday notes

Def

The dimension of a subspace is the number of basis vectors

Ex: a) Standard basis for $\mathbb{R}^3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

$$\dim(\mathbb{R}^3) = 3$$

b) $\dim(\mathbb{R}^n) = n$

c) $\dim(\text{plane}) = 2$

d) $\dim(\text{line}) = 1$

Def

The rank of a matrix is the # of leading entries in REF/RREF

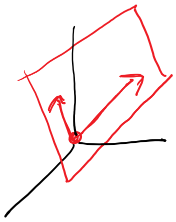
Rephrased: $\text{rank} = \dim[\text{row}(A)] = \dim[\text{col}(A)]$

Def

$\boxed{\text{The } \underline{\text{nullity}} \text{ of a matrix is the } \# \text{ of parameters in solution to } A\vec{x}=\vec{0}}$

<u>Rephrased</u>: $\text{nullity} = \dim[\text{null}(A)]$

<u>Quick Ex</u>:



null (A)

$\text{nullity}(A) = 2$

<u>Ex</u>: $A = \begin{bmatrix} 1 & 5 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

Find the rank and nullity

$\rightsquigarrow \begin{bmatrix} 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ RREF

$\text{rank} = 3$

nullity: $\left[\begin{array}{cccc|c} 1 & 5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

$\text{nullity} = 1$

Fact

$\boxed{\begin{array}{l} \text{For any matrix } A, \\ \quad \text{rank} + \text{nullity} = \#\text{columns in } A \end{array}}$

Intuitively:

$\boxed{\begin{array}{c}\#\text{columns} \\ \text{with circles}\end{array}} + \boxed{\begin{array}{c}\#\text{columns} \\ \text{without} \\ \text{circles}\end{array}} = \#\text{columns}$

Summary

Subspaces Associated with A	Name for the Dim. of Subspace	How to Find Dim. of Subspace
row space	rank	# leading entries in REF/RREF
column space	rank	# leading entries in REF/RREF
null space	nullity	#columns - rank

Handout

"Long Fundamental Theorem"

For any given matrix,
all IS statements are true
or " false

$\boxed{a-e}$ Section 3.3

$\boxed{f, g}$

$\boxed{h-j}$ Info about columns of A

$\boxed{k-m}$ " rows "

$\boxed{n, o}$ Ch 4

Ex: Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 1 & 1 & 4 \end{bmatrix}$$

m) is true exactly when f) is true

$$\boxed{\begin{array}{ll} \text{If } \operatorname{rank}(A)=3, & \text{yes} \\ \text{If } \operatorname{rank}(A)\neq 3, & \text{no} \end{array}}$$

$$A \rightsquigarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \text{ REF}$$

$$\operatorname{rank}(A) = 3$$

$$\boxed{\text{Yes}}$$