Week 6 Friday

February 14, 2020 9:56 AM

3.5 Subspaces and Dimension Cont'd

Ch 7: Best-fit curve through a set of points

Need column space and null space

Ex: $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 0 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

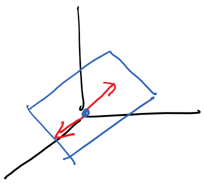
RREF of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ RREF of $A^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Find a basis for:

a) row(A)

Use nonzero rows of REF/RREF

$\{ [1 \ 0 \ 0], [0 \ 1 \ 1] \}$

row(A) 

Basis for row(A) = $\{ [1 \ 0 \ 0], [0 \ 1 \ 1] \}$

b) col(A)

Use the leading entries of RREF as a guide

$\begin{bmatrix} 1 & & \\ & 1 & \\ & & \end{bmatrix}$

Use Columns 1 and 2 of A

basis for col(A) = $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

- Row ops change the column space (can't use RREF)
- Columns don't move around during row ops

c) row(A), consisting of rows of A

row(A) = col(A^T)

Find a basis for col(A^T)

RREF of $A^T = \begin{bmatrix} 1 & & 1 \end{bmatrix}$

Use Columns 1 and 3 of A^T

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

or $\{ [1 \ 1 \ 1], [1 \ 0 \ 0] \}$

Ex: $A = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 8 & 12 \end{bmatrix}$

Find a basis for null(A)

null(A) = $\{ \vec{x}$ such that $A\vec{x} = \vec{0} \}$

Solve $A\vec{x} = \vec{0}$

Each parameter gives a basis vector

$\left[\begin{array}{ccc|c} 1 & 4 & 6 & 0 \\ 2 & 8 & 12 & 0 \end{array}\right]$

$\rightsquigarrow \begin{array}{cccc} x_1 & x_2 & x_3 & \\ \left[\begin{array}{c} 1 \\ 0 \end{array}\right. & \begin{array}{c} 4 \\ 0 \end{array} & \begin{array}{c} 6 \\ 0 \end{array} & \left|\begin{array}{c} 0 \\ 0 \end{array}\right] \end{array}$ RREF

$\boxed{x_2 = s}$ $\boxed{x_3 = t}$

$x_1 + 4x_2 + 6x_3 = 0 \rightarrow \boxed{x_1 = -4s - 6t}$

$\vec{x} = \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} t$

Basis for null (A) = $\left\{ \begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} \right\}$

Ex: Find a basis for span$\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 24 \end{bmatrix} \right)$

Method I $A = \begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix}$

Find a basis for row(A)

Method II $A = \begin{bmatrix} | & | & | \end{bmatrix}$

Method II

$A = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

Find a basis for Col(A)

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 6 \\ 1 & 5 & 24 \end{bmatrix}$$

Find a basis for row(A)

$$\rightsquigarrow \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{RREF}$$

Basis = $\{ [1 \ 0 \ -6], [0 \ 1 \ 6] \}$

Def

Given a basis $\mathcal{B} = \{ \vec{v}_1, \vec{v}_2, \dots, \vec{v}_n \}$ of $\mathbb{R}^n$, the coordinate vector of $\vec{v}$ is

$$[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} \quad \text{such that} \quad \vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$$

Ex: Find $[\vec{v}]_{\mathcal{B}}$ for $\vec{v} = \begin{bmatrix} 5 \\ 15 \\ 28 \end{bmatrix}$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$$

$$\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 5 & 1 & 15 \\ 3 & 6 & 4 & 28 \end{array}\right]$$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad \text{RREF}$$

$c_1 = -2$
$c_2 = 3$
$c_3 = 4$

Therefore $[\vec{v}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}$