Week 6 Thursday

February 13, 2020 9:40 AM

3.5 Subspaces and Dimension Cont'd

Fact

A set of vectors S is a subspace of $\mathbb{R}^n$ if and only if S is the span of some vectors.

3 Subspaces Associated with a Matrix

row space of A : span of the rows of A
Written $\text{row}(A)$

Column space of A : span of columns of A
Written $\text{col}(A)$

null space of A : $\{\vec{x}$ such that $A\vec{x}=\vec{0}\}$
Written $\text{null}(A)$

Ex: $A=\begin{bmatrix}1&2&0\\1&2&1\end{bmatrix}$

a) Is $\begin{bmatrix}6\\10\end{bmatrix}$ in $\text{col}(A)$?

$c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}2\\2\end{bmatrix}+c_3\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}6\\10\end{bmatrix}$?

$0\begin{bmatrix}1\\1\end{bmatrix}+3\begin{bmatrix}2\\2\end{bmatrix}+4\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}6\\10\end{bmatrix}$

$\boxed{\text{Yes}}$

b) Is $[1\ 2\ 5]$ in $\text{row}(A)$?

$[1\ 2\ 5]=c_1[1\ 2\ 0]+c_2[1\ 2\ 1]$?

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c}1&1&1\\2&2&2\\0&1&5\end{array}\right]$$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\rightsquigarrow\left[\begin{array}{cc|c}1&0&-4\\0&1&5\\0&0&0\end{array}\right] \quad \text{RREF}$$

$c_1=-4$
$c_2=5$

$\boxed{\text{Yes}}$

$-4[1\ 2\ 0]+5[1\ 2\ 1]=[1\ 2\ 5]$

c) Is $\begin{bmatrix}1\\2\\3\end{bmatrix}$ in $\text{null}(A)$?

$A\vec{x}=\vec{0}$?

$\begin{bmatrix}1&2&0\\1&2&1\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}\neq\vec{0}$

$\boxed{\text{No}}$

Def

A set $\mathcal{B}$ is a basis for a subspace S if :

1) $\text{span}(\mathcal{B}) = S$

2) $\mathcal{B}$ is linearly independent

Intuitively: Set $\mathcal{B}$ contains enough vectors to describe S and set $\mathcal{B}$ has no redundancy.

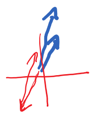
$\vec{v}_1, \vec{v}_2$ are lin. ind. if

$c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{0}$

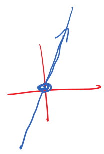
$\rightarrow \quad 0\vec{v}_1 + 0\vec{v}_2 = \vec{0}$

Ex: a) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$

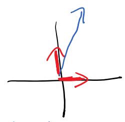
b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ "



c) $\left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$ is not a basis for $\mathbb{R}^2$



d) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ is not a basis for $\mathbb{R}^2$



(not lin. ind.)

Ex: $A = \begin{bmatrix} 2 & 3 & 7 \\ 4 & 7 & 10 \\ 8 & 17 & 8 \end{bmatrix}$

Find a basis for :

a) row(A)

Use nonzero rows of REF/RREF

$\rightsquigarrow \begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix}$ REF

Basis for row(A) $= \{ [2 \ 3 \ 7], [0 \ 1 \ -4] \}$

b) col(A)

Use columns of A corresponding to the leading entries of REF/RREF

$\begin{matrix} \rightarrow \\ \rightarrow \end{matrix} \begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & -4 \end{bmatrix}$ REF

$$\rightarrow \begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix} \text{ REF}$$

Use Columns 1 and 2 of A

Basis for col(A) = $\left\{ \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ 17 \end{bmatrix} \right\}$

Caution: Row operations change the column space.
(Don't use columns of REF/RREF)

e.g. $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ is not in col(A)

c) row(A) consisting of rows of A

DIFFERENT FROM PART a)

row(A) = col(A^T)

$$A^T = \begin{bmatrix} 2 & 4 & 8 \\ 3 & 7 & 17 \\ 7 & 10 & 8 \end{bmatrix}$$

$$\rightsquigarrow \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \text{ REF}$$

Find a basis for col(A^T)

Use Columns 1 and 2 of A^T

$$\left\{ \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix} \right\} \checkmark$$

or $\{ [2\ 3\ 7], [4\ 7\ 10] \}$

A basis is not unique.