

3.4 LU Factorization Cont'd

Ex: Find LU and use it to solve

$$\begin{bmatrix} 2 & -4 & 0 \\ 3 & -1 & 4 \\ -1 & 2 & 2 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}$$

① Find L and U

$$\begin{array}{l} R_2 - \frac{3}{2}R_1 \\ R_3 + \frac{1}{2}R_1 \end{array} \begin{bmatrix} 2 & -4 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{bmatrix} \begin{array}{l} k = \frac{3}{2} \\ k = -\frac{1}{2} \end{array}$$

$$\begin{aligned} 3 + ?(2) &= 0 \\ ? &= -\frac{3}{2} \end{aligned}$$

$$R_3 + 0R_2 \begin{bmatrix} 2 & -4 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 2 \end{bmatrix} k=0$$

REF

$$U = \text{REF}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

k-values

$$A = LU$$

②

$$A\vec{x} = \vec{b}$$

$$LU\vec{x} = \vec{b}$$

\vec{y}

Solve $L\vec{y} = \vec{b}$

$$\begin{array}{ccc|c} y_1 & y_2 & y_3 & \\ \hline 1 & 0 & 0 & 2 \\ \frac{3}{2} & 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 & -5 \end{array}$$

start @ bottom

Start @ top

$$\boxed{y_1 = 2}$$

$$\frac{3}{2}y_1 + y_2 = 0 \rightarrow \boxed{y_2 = -3}$$

$$-\frac{1}{2}y_1 + y_3 = -5 \rightarrow \boxed{y_3 = -4}$$

$$\vec{y} = \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix}$$

③

Solve $U\vec{x} = \vec{y}$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & -4 & 0 & 2 \\ 0 & 5 & 4 & -3 \\ 0 & 0 & 2 & -4 \end{array}$$

Start @ bottom

$$2x_3 = -4 \rightarrow \boxed{x_3 = -2}$$

$$5x_2 + 4x_3 = -3 \rightarrow \boxed{x_2 = 1}$$

$$2x_1 - 4x_2 = 2 \rightarrow \boxed{x_1 = 3}$$

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

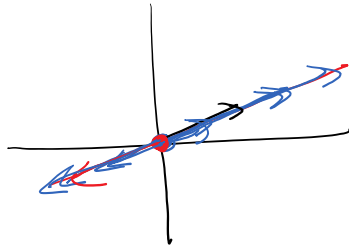
3.5 Subspaces and Dimension

Def

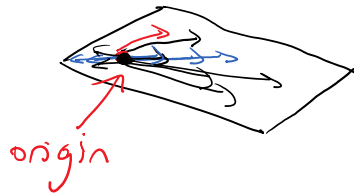
A subspace of \mathbb{R}^n is a collection of vectors S such that:

- 1) $\vec{0}$ is in S
- 2) If \vec{u} is in S , then $c\vec{u}$ is in S (c : any real #)
- 3) If \vec{u} and \vec{v} are in S , then $\vec{u} + \vec{v}$ is in S

Ex: A line through the origin
is a subspace of \mathbb{R}^2



Ex: A plane through the origin
is a subspace of \mathbb{R}^3



Ex: Subspace of \mathbb{R}^3 ?

$$S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x = z + 1 \right\}$$

$$S = \left\{ \begin{bmatrix} z+1 \\ y \\ z \end{bmatrix} \right\}$$

NO $\vec{0}$ is not in S .

Yes if all 3 properties are true
No if ≥ 1 property is false

Ex: Subspace of \mathbb{R}^2 ?

$$S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid y = x^3 \right\}$$

$$S = \left\{ \begin{bmatrix} x \\ x^3 \end{bmatrix} \right\}$$

$$S = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \end{bmatrix}, \begin{bmatrix} -4 \\ -64 \end{bmatrix}, \dots \right\}$$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is in S

$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ is not in S

Property 2 fails

No

To show that a property is false, can use numbers
" " true, must use variables

Ex: Show that $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \mid 3x - 4y = 0 \right\}$
is a subspace of \mathbb{R}^2

$$3x - 4y = 0$$

$$3x = 4y$$

$$x = \frac{4}{3}y$$

$$S = \left\{ \begin{bmatrix} \frac{4}{3}y \\ y \end{bmatrix} \right\}$$

1) $\vec{0}$ is in S ✓

($\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has the correct form)

1) $\vec{0}$ is in S ✓ $\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ has the correct form} \right)$

2) Let $\vec{u} = \begin{bmatrix} \frac{4}{3}a \\ a \end{bmatrix}$ be in S

$c\vec{u} = \begin{bmatrix} \frac{4}{3}ca \\ ca \end{bmatrix}$ is in S (correct form) ✓

3) Let $\vec{u} = \begin{bmatrix} \frac{4}{3}a \\ a \end{bmatrix}$ $\vec{v} = \begin{bmatrix} \frac{4}{3}b \\ b \end{bmatrix}$

$$\vec{u} + \vec{v} = \begin{bmatrix} \frac{4}{3}a + \frac{4}{3}b \\ a + b \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{3}(a+b) \\ a+b \end{bmatrix} \text{ is in } S \text{ (correct form)} \checkmark$$