Week 6 Tuesday
February 11, 2020 8:47 AM

Test 2 FRI MAR 6
2.3-2.4, 3.1-3.5

3.3 Cont'd

Handout: "Short Fundamental Theorem"

For a given $n \times n$ matrix,
all 5 statements are true or
" false

Ex: $A = \begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$

A^{-1} exists $(\det A \neq 0)$

a)
b)
c)
d)
e)
all true

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

A^{-1} is undefined $(\det A = 0)$

a) - e) are all false

3.4 LU Factorization

An upper triangular matrix has 0's below diagonal

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

A unit lower triangular matrix has 0's above diagonal
and 1's on diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 5 & 1 \end{bmatrix}$$

$A = LU$ "LU Factorization"

L: unit lower Δ

U: upper Δ

e.g. $$\begin{bmatrix} 2 & 1 & 1 \\ 4 & 4 & 3 \\ 8 & 10 & 13 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

Ex: Solve $$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \end{array}$$ $$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 4 & 4 & 3 & 2 \\ 8 & 10 & 13 & -8 \end{array}\right]$$

using LU above

① $$A\vec{x} = \vec{b}$$

$$L\underbrace{U\vec{x}}_{\vec{y}} = \vec{b}$$

Solve $L\vec{y} = \vec{b}$

$$\begin{array}{ccc|c} y_1 & y_2 & y_3 & \\ \end{array}$$ $$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 2 \\ 4 & 3 & 1 & -8 \end{array}\right]$$

Start @ top

$$y_1 = 1$$

$$\underset{2}{\cancel{2y_1}} + y_2 = 2 \rightarrow y_2 = 0$$

$$\underset{4}{\cancel{4y_1}} + \cancel{3y_2} + y_3 = -8 \rightarrow y_3 = -12$$

$$\vec{y} = \begin{bmatrix} 1 \\ 0 \\ -12 \end{bmatrix}$$

② Solve $U\vec{x} = \vec{y}$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 6 & -12 \end{array}\right]$$

Start @ bottom

$6x_3 = -12 \rightarrow x_3 = -2$

$2x_2 + \underset{-2}{\cancel{x_3}} = 0 \rightarrow x_2 = 1$

$2x_1 + \underset{1}{\cancel{x_2}} + \underset{-2}{\cancel{x_3}} = 1 \rightarrow x_1 = 1$

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Finding LU

$A \rightsquigarrow$ REF using only (current row) − k (pivot row)

Record the k-value

(Only possible when no row swaps are required)

Ex: Find LU for $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 4 & 3 \\ 8 & 10 & 13 \end{bmatrix}$

$$\begin{array}{r} R_2 - 2R_1 \\ R_3 - 4R_1 \end{array} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 6 & 9 \end{bmatrix} \begin{array}{l} k=2 \\ k=4 \end{array}$$

$$R_3 - 3R_2 \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 6 \end{bmatrix} k=3$$

U = REF

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & \underline{\underline{3}} & 1 \end{bmatrix}$$

k-values

Why This Works

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Elementary matrix for $R_2 \to R_2 - 2R_1$ $\quad E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

L <u>undoes</u> the operations that turn A into U

L turns U into A

$$LU = A$$