Week 5 Wednesday

February 5, 2020 9:04 AM

3.2 Matrix Algebra

Ex: Is $\begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix}$ a linear combination of

$\begin{bmatrix} 0 & 1 \\ 6 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix}$?

Let $$\underbrace{c_1 \begin{bmatrix} 0 & 1 \\ 6 & 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix}}_{\text{linear combination}} = \begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix}$$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 6 & 7 & 4 \\ 2 & 2 & 2 \end{array}\right]$$

Reorder rows $$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 6 & 7 & 4 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{c} R_2 - 6R_1 \\ R_3 - 2R_1 \end{array} \quad \begin{array}{ccc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \quad \text{REF} \checkmark$$

Consistent system

$\boxed{\text{Yes}}$

To check: $c_2 = -2$

$c_1 + c_2 = 1 \rightarrow c_1 = 3$

$$3 \begin{bmatrix} 0 & 1 \\ 6 & 2 \end{bmatrix} - 2 \begin{bmatrix} 0 & 1 \\ 7 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix} \checkmark$$

Ex: Find the general form of

$$\text{span}\left(\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \right)$$

span = set of all linear combinations e.g. span $(\vec{v}_1, \vec{v}_2) = \{ c_1\vec{v}_1 + c_2\vec{v}_2 \}$

Section 2.3

Let $$c_1 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

Want conditions on w, x, y, z

$$\begin{array}{ccc|c} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & w \\ 0 & 0 & 1 & x \\ 2 & 6 & 4 & y \\ 0 & 1 & 5 & z \end{array}\right]$$

Each zero row of REF will give a condition.

$$R_3 - 2R_1 \quad \left[\begin{array}{ccc|c} 1 & 3 & 2 & w \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & y-2w \\ 0 & 1 & 5 & z \end{array}\right]$$

$$\text{Reorder rows} \quad \left[\begin{array}{ccc|c} 1 & 3 & 2 & w \\ 0 & 1 & 5 & z \\ 0 & 0 & 1 & x \\ 0 & 0 & 0 & y-2w \end{array}\right] \quad \text{REF} \checkmark$$

Consistent system

$$\Longrightarrow \quad y - 2w = 0$$

General form $= \left\{ \begin{bmatrix} w & x \\ y & z \end{bmatrix} \text{ such that } \cancel{y-2w=0} \; y = 2w \right\}$

$$= \left\{ \begin{bmatrix} w & x \\ 2w & z \end{bmatrix} \right\}$$

Follow Up:

$\begin{bmatrix} 3 & 7 \\ 6 & -9 \end{bmatrix}$ <u>is</u> in the span

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is not in the span

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<u>Ex</u>: Find the general form of

$$\text{span}\left(\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \right)$$

let $\quad c_1 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + c_2 \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

$$\begin{array}{cc|c} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & x \\ 2 & 4 & y \\ 0 & 5 & z \end{array}\right]$$

$$R_3 - 2R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & x \\ 0 & 0 & y-2w \\ 0 & 5 & z \end{array}\right]$$

$$R_4 - 5R_2 \quad \left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & x \\ 0 & 0 & y-2w \\ 0 & 0 & z-5x \end{array}\right] \qquad \text{REF}$$

Each zero row of REF gives a condition.

Consistent system $\Rightarrow$ $\cancel{y-2w=0}$ and $\cancel{z-5x=0}$

$y = 2w$ $\qquad$ $z = 5x$

General form $= \left\{ \begin{bmatrix} w & x \\ 2w & 5x \end{bmatrix} \right\}$

"the set of all matrices that look like this"

e.g. $\begin{bmatrix} 1 & 1 \\ 2 & 5 \end{bmatrix}$ is in the span

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is not "

$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ is not "

Ex: Are $\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 6 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ linearly independent?

Let $c_1 \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + c_2 \begin{bmatrix} \ \end{bmatrix} + c_3 \begin{bmatrix} \ \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 6 & 4 & 0 \\ 0 & 1 & 5 & 0 \end{array}\right]$$

1 solution ($c_1 = c_2 = c_3 = 0$) $\rightarrow$ $\boxed{\text{Yes}}$ (Linearly independent)

∞-many solutions $\rightarrow$ $\boxed{\text{No}}$ (Linearly dependent)

(Linearly independent)

(Linearly dependent)

$$\rightsquigarrow \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{REF}$$

1 solution

$\boxed{\text{Yes}}$

$c_3 = 0$

$c_2 + 5c_3 = 0 \Rightarrow c_2 = 0$

$c_1 = 0$

Overkill

Aside

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \infty\text{-many}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & \\ 0 & 1 & 3 & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & \end{array}\right] \quad \infty\text{-many}$$