Week 5 Tuesday

February 4, 2020 10:21 AM

3.1 Matrix Operations Cont'd

Ex: Write as a matrix equation

$$\begin{cases} 7x - 3y = 12 \\ -8x + 9y = -2 \end{cases}$$

$A\vec{x} = \vec{b}$

- A: Coefficients
- $\vec{x}$: Variables (column)
- $\vec{b}$: Constants (column)

$$\begin{bmatrix} 7 & -3 \\ -8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -2 \end{bmatrix}$$

Ex: A: test marks

	Al	Bob
T1	50	60
T2	90	80
Exam	75	70

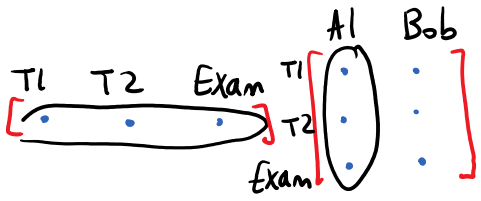
B: weightings

T1	T2	Exam
0.2	0.2	0.6

Find Al and Bob's final grades

→ Need compatible sizes and categories

$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix}$ undefined



$$= \begin{bmatrix} 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 50 & 60 \\ 90 & 80 \\ 75 & 70 \end{bmatrix}$$

$$= \begin{matrix} Al & Bob \\ \end{matrix} \begin{bmatrix} 73 & 70 \end{bmatrix}$$

Ex: Find 2nd column of

$$\begin{bmatrix} 1 & 2 \\ 2 & 6 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 4 & 7 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 16 \\ 46 \\ 65 \end{bmatrix}$$

Notice $\begin{bmatrix} 16 \\ 46 \\ 65 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + 7\begin{bmatrix} 2 \\ 6 \\ 9 \end{bmatrix}$

Fact Columns of AB are linear combinations of the columns of A

Useful in Section 7.3

Powers of a Matrix

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$A^2 = AA$$

$$= \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$$

Usual exponent rules apply

$A^3 = A^2 A$ or $A^3 = A A^2$

$A^5 A^8 = A^{13}$

$(A^4)^7 = A^{28}$

Fact: $AI = A$, $IA = A$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ etc.

Analogy to $7(1) = 7$

Ex: Simplify B^{2018} given $B^3 = I$.

$2018 = ?(3) + r$

$\frac{2018}{3} \approx 672.7$

$2018 = 672(3) + r$

$2018 - 672(3) = r$

$2018 = 672(3) + 2$

B^{2018}

$= B^{672(3)+2}$

$= B^{672(3)} B^2$

$= (B^3)^{672} B^2$

$= I^{672} B^2$

$= IB^2$

$= B^2$

Def

Let $\mathbf{0}$ be the zero matrix

$$\mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{or} \quad \mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{etc.}$$

Ex: Find a 2x2 matrix A so that $A^2 = \mathbf{0}$ but $A \neq \mathbf{0}$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \checkmark$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$