3.1 Matrix Operations

Size of a matrix : (# rows) x (# columns)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \text{is} \quad 2 \times 3$$

Entry of a matrix : $a_{23} = 6$ or $[A]_{23} = 6$

Square matrix : has size 2x2, 3x3 etc.

Identity matrix : $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc.

Diagonal matrix : $D = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

Ex: $A = \begin{bmatrix} 1 & 6 & 1 \\ -2 & -2 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & -3 \\ 1 & 6 & 9 \end{bmatrix}$

Find:

a) $A+B$

$$= \begin{bmatrix} 2 & 6 & -2 \\ -1 & 4 & 13 \end{bmatrix}$$

A+B is undefined if A, B have different sizes

b) $3A$

$$= \begin{bmatrix} 3 & 18 & 3 \\ -6 & -6 & 12 \end{bmatrix}$$

"scalar multiplication"

c) $A - 3B$

$$= A + (-3B)$$

$$= \begin{bmatrix} 1 & 6 & 1 \\ -2 & -2 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 0 & 9 \\ -3 & -18 & -27 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 6 & 10 \\ -5 & -20 & -23 \end{bmatrix}$$

Def

The <u>transpose</u> of A, written A^T, has rows and columns interchanged.

A is <u>symmetric</u> if $A^T = A$

<u>Ex</u>:

a) $A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 6 & 3 \\ 4 & 3 & -1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 6 & 3 \\ 4 & 3 & -1 \end{bmatrix}$

A is symmetric

b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$A^T \neq A$

A is not symmetric

c) $B = \begin{bmatrix} \vdots & \vdots & \vdots \end{bmatrix}$

B is not symmetric

B is 2×3

B^T is 3×2

Matrix Multiplication

$$AB = \begin{bmatrix} r_1 \cdot c_1 & r_1 \cdot c_2 & \\ r_2 \cdot c_1 & & \\ & \text{etc} & \end{bmatrix}$$

where $r_i = i^{th}$ row of A

$c_j = j^{th}$ column of B

Ex:

$$\begin{bmatrix} 1 & 4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 9 & 27 \\ -2 & 0 & 0 \end{bmatrix}$$

$[1 \ \ 4] \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1(1) + 4(0)$

$[1 \ \ 4] \cdot \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$[-2 \ \ 1] \cdot \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Consider the sizes:

$$(2\times2)(2\times3) = 2\times3$$

Inside numbers must be equal, otherwise AB is undefined.

Outside numbers give the size of AB

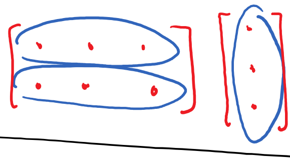
Ex: A is 2×3

B is 3×1

Size of AB and BA?

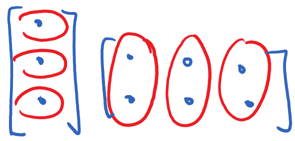
AB: $(2\times 3)(3\times 1)$ ✓

AB is 2×1



BA: $(3\times 1)(2\times 3)$ $\neq$

BA is undefined



Ex: Find BC and CB

$$B=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}\qquad C=\begin{bmatrix}1&-1\\2&4\end{bmatrix}$$

$$BC=\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}\begin{bmatrix}1&-1\\2&4\end{bmatrix}$$

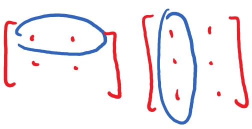
$$=\begin{bmatrix}5&7\\11&13\\17&19\end{bmatrix}$$

$[1\ \ 2]\cdot\begin{bmatrix}-1\\4\end{bmatrix}$

$[5\ \ 6]\cdot\begin{bmatrix}1\\2\end{bmatrix}$

CB is undefined

$(2\times 2)(3\times 2)$ $\neq$



$\boxed{\text{Fact}\quad AB\neq BA \text{ in general}}$

Why We Multiply Like This

Consider

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1x + 2y \\ 3x + 4y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\begin{cases} 1x + 2y = 5 \\ 3x + 4y = 6 \end{cases}$$

So that we can describe and solve systems

System: $A\vec{x} = \vec{b}$

Coefficients

variables (column)

Constants (column)