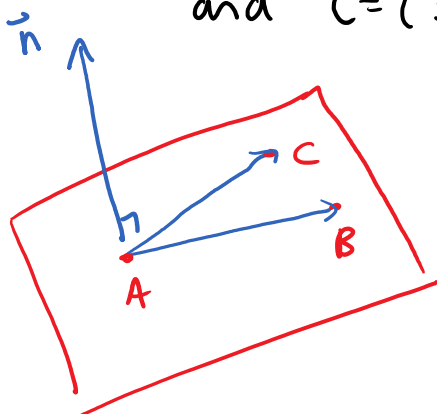Bring music / earplugs

Test Review
_____

$\underline{Ex}$: Find normal form of the plane
through $A = (1, 2, 3)$, $B = (6, 5, 4)$
and $C = (3, -3, -3)$.

$\vec{AB} = [5, 3, 1]$ Think B-A

$\vec{AC} = [2, -5, -6]$

$\vec{n} = \vec{AB} \times \vec{AC}$

$= [-13, 32, -31]$

$\begin{matrix} 5 & 3 & 1 & 5 & 3 \\ 2 & -5 & -6 & 2 & -5 \end{matrix}$

Normal form $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$\begin{bmatrix} -13 \\ 32 \\ -31 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ 32 \\ -31 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
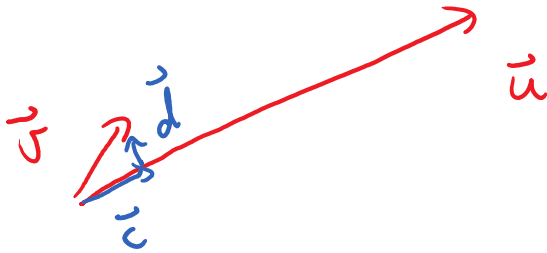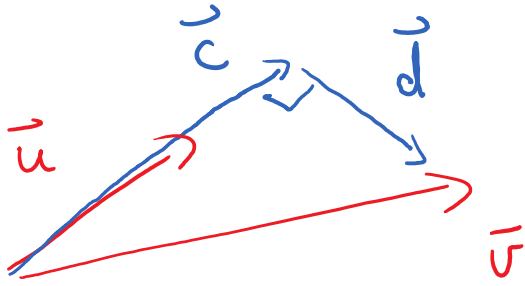
$\underline{Ex}$: Let $\vec{u} = [1, -1, 2]$ and $\vec{v} = [3, 0, 4]$
Find $\vec{c}$ and $\vec{d}$ so that:
$\vec{c}$ is parallel to $\vec{u}$,
$\vec{d}$ " perpendicular to $\vec{u}$,
and $\vec{v} = \vec{c} + \vec{d}$

$$\vec{c} = proj_{\vec{u}}\,\vec{v}$$

$$= \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\,\vec{u}$$

$$= \frac{11}{6}\,[1, -1, 2]$$

$$\vec{v} = \vec{c} + \vec{d}$$

$$\vec{d} = \vec{v} - \vec{c}$$

$\times\frac{6}{6}$

$$= [3, 0, 4] - \frac{11}{6}[1, -1, 2]$$

$$= \frac{1}{6}[18, 0, 24] - \frac{1}{6}(11, -11, 22)$$

$$= \frac{1}{6}[7, 11, 2]$$

Ex: Volume of parallelepiped formed by $\bar{u} = [-4, 2, 3]$, $\bar{v} = [2, 1, 2]$ and $\bar{w} = [3, -3, 6]$?

$$V = \text{abs. value of} \begin{vmatrix} -4 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & -3 & 6 \end{vmatrix}$$

$$= \left| -4 \begin{vmatrix} 1 & 2 \\ -3 & 6 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 3 & 6 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} \right|$$

$$\begin{bmatrix} + & - & + \end{bmatrix}$$

$$= |\ -4(12) - 2(6) + 3(-9)\ |$$

$$= |\ -87\ |$$

$$= 87$$

Ex: Solve by Gauss-Jordan Elimination

$$\begin{bmatrix} 1 & -2 & 3 & | & 6 \\ 0 & 11 & -8 & | & -11 \\ 0 & 22 & -16 & | & -22 \end{bmatrix}$$

$$\frac{R_2}{11} \quad \begin{bmatrix} 1 & -2 & 3 & | & 6 \\ 0 & \textcircled{1} & -\frac{8}{11} & | & -1 \\ 0 & 22 & -16 & | & -22 \end{bmatrix}$$

$$R_1 + 2R_2 \qquad \begin{matrix} x & y & z \end{matrix}$$

$$3 - \frac{16}{11} = \frac{33}{11} - \frac{16}{11}$$

$$R_1 + 2R_2 \qquad \begin{bmatrix} \textcircled{1} & 0 & \frac{17}{11} & | & 4 \\ 0 & \textcircled{1} & -\frac{8}{11} & | & -1 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$R_3 - 22R_2$$

RREF

$$\boxed{z = t}$$

$$x + \frac{17}{11}z = 4$$

$$\boxed{x = 4 - \frac{17}{11}t}$$

$$\boxed{y = -1 + \frac{8}{11}t}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{17}{11} \\ \frac{8}{11} \\ 1 \end{bmatrix} t$$