Week 4 Tuesday
January 28, 2020 8:36 AM

2.3 Span and Linear Independence Cont'd

Ex: Are $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\2\end{bmatrix}$ linearly independent?

Let $c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = \vec{0}$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$$

$R_2 - R_1$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$

$\dfrac{R_2}{-1}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right]$

$R_1 - R_2$, $R_3 + R_2$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \quad \text{REF}$$

$c_1 = 0$

$c_2 = 0$

$2c_3 = 0 \rightarrow c_3 = 0$

$\boxed{\text{Yes}}$

Recall

$\boxed{\text{Vectors are lin. ind. if } c_1 = c_2 = c_3 = 0 \text{ is the only solution.}}$

Fact

A set of $>n$ vectors in $\mathbb{R}^n$ is linearly **dependent**

Ex: $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 7 \end{bmatrix}$ are lin. **dependent**

Why? Let $c_1\vec{v}_1 + \dots + c_m\vec{v}_m = \vec{0}$

$$n\left\{ \begin{array}{c} \begin{array}{ccc} c_1 & \cdots & c_m \end{array} \\ \underbrace{\begin{bmatrix} & & \\ & \vdots & \\ & & \end{bmatrix}}_{>n} \end{array} \right. \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

System has ∞-many solutions (Section 2.2)

$\Rightarrow$ Vectors are lin. **dependent**

Ex: Find a linear dependence relationship (linear dependency) among

$$\begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 30 \end{bmatrix}$$

Method I Let $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

$$\begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 6 & 6 & 30 & 0 \end{array}\right] \end{array}$$

$$R_2 - 6R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & -6 & 6 & 0 \end{array}\right]$$

$$\frac{R_2}{-6} \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ R_1 - 2R_2 \end{array} \quad \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$

$$R_1 - 2R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 6 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \quad \text{RREF}$$

$$\boxed{c_3 = t}$$

$$c_1 + 6c_3 = 0 \rightarrow \quad \boxed{c_1 = -6t}$$

$$\boxed{c_2 = t}$$

Choose any <u>nonzero</u> t-value : $\quad t = 1$

$$c_1 = -6$$

$$c_2 = 1 = c_3$$

$$c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 = \vec{0}$$

$$\boxed{-6\bar{v}_1 + \bar{v}_2 + \bar{v}_3 = \bar{0}}$$

Method II

Vectors $\to$ rows

$$\begin{array}{c} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \end{array} \begin{bmatrix} 1 & 6 \\ 2 & 6 \\ 4 & 30 \end{bmatrix}$$

$$\begin{array}{c} \vec{v}_1 \\ \vec{v}_2 - 2\vec{v}_1 \\ \vec{v}_3 - 4\vec{v}_1 \end{array} \begin{bmatrix} 1 & 6 \\ 0 & -6 \\ 0 & 6 \end{bmatrix}$$

$$\underbrace{(\vec{v}_3 - 4\vec{v}_1) + (\vec{v}_2 - 2\vec{v}_1)}_{-6\vec{v}_1 + \vec{v}_2 + \vec{v}_3} \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\boxed{-6\vec{v}_1 + \vec{v}_2 + \vec{v}_3 = \vec{0}}$$

Any zero row gives a linear dependency.

Method I gives the <u>general</u> dependency

Method II gives 1 <u>specific</u> dependency

Preview of Section 3.5

The smallest set of vectors that describe a geometric object?

Need

1) span (vectors) = object
2) vectors to be linearly independent (no redundancy)

2.4 Applications

Ex: Find the parabola through $(1,-2)$, $(-1,8)$ and $(2,-1)$

Parabola $y = ax^2 + bx + c$

Find a, b, c

$x = 1$
$y = -2$:

$$-2 = a + b + c \quad (1)$$

$$8 = a - b + c \quad (2)$$

$$-1 = 4a + 2b + c \quad (3)$$

$$\begin{array}{ccc} a & b & c & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ 1 & -1 & 1 & 8 \\ 4 & 2 & 1 & -1 \end{array}\right]$$

$$\rightsquigarrow \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array} \quad \text{RREF}$$

$a = 2 \qquad b = -5 \qquad c = 1$

$$y = ax^2 + bx + c$$

$$\boxed{y = 2x^2 - 5x + 1}$$