Week 4 Monday

January 27, 2020 7:54 AM

Test Friday

1.1-1.3, Cross Product, 2.1-2.2 (6 Questions)

No Formula Sheet

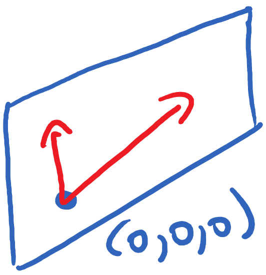
Practice Problems www.leahhoward.com

No Office Hours Wednesday

Review Thursday

2.3 Span and Linear Independence

Ex: Find an equation for span $\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \right)$



Plane through the origin

Vector Form

$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

$\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$

or

General Form

$\vec{n} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \\ 3 \end{bmatrix}$

$-3x - 5y + 3z = d$

Sub $(0,0,0)$: $0 = d$

$\boxed{-3x - 5y + 3z = 0}$

Ex: a) Show that $\text{span}\left(\begin{bmatrix}1\\3\end{bmatrix},\begin{bmatrix}2\\1\end{bmatrix}\right)=\mathbb{R}^2$

Let $\underbrace{c_1\begin{bmatrix}1\\3\end{bmatrix}+c_2\begin{bmatrix}2\\1\end{bmatrix}}_{\text{span}}=\begin{bmatrix}a\\b\end{bmatrix}$ any vector in $\mathbb{R}^2$

Show the system has a solution.

$$\begin{array}{cc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & a \\ 3 & 1 & b \end{array}\right]$$

$$R_2-3R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & -5 & b-3a \end{array}\right] \quad \text{REF}$$

System has a solution ✓

b) Write $\begin{bmatrix}a\\b\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\3\end{bmatrix}$ and $\begin{bmatrix}2\\1\end{bmatrix}$.

Let $c_1\begin{bmatrix}1\\3\end{bmatrix}+c_2\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}a\\b\end{bmatrix}$

$\vdots$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & -5 & b-3a \end{array}\right]$$

$$\frac{R_2}{-5} \quad \left[\begin{array}{cc|c} 1 & 2 & a \\ 0 & 1 & -\frac{1}{5}(b-3a) \end{array}\right]$$

$$R_1 - 2R_2 \quad \begin{array}{ccc} c_1 & c_2 & \\ \hline \end{array}\left[\begin{array}{cc|c} 1 & 0 & \frac{2b-a}{5} \\ 0 & 1 & \frac{3a-b}{5} \end{array}\right] \quad \text{RREF}$$

$$\boxed{\begin{aligned} & a + \frac{2}{5}(b-3a) \\ & = -\frac{1}{5}a + \frac{2}{5}b \end{aligned}}$$

$$c_1 = \frac{2b-a}{5} \qquad c_2 = \frac{3a-b}{5}$$

To check: $c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

$$\frac{2b-a}{5} \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \frac{3a-b}{5} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \checkmark$$

1st component: $\left(\frac{2b-a}{5}\right)(1) + \left(\frac{3a-b}{5}\right)(2) = a$? Yes

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Is the system solvable? REF

Solve the system RREF

Def

Given $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$, Consider solutions to

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n = \vec{0}$$

$c_1 = c_2 = \dots = c_n = 0$ is the only solution $\longrightarrow$ Vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ are linearly independent

$c_1 = c_2 = \dots = c_n = 0$ and other solutions $\longrightarrow$ Vectors $\vec{v}_1, \dots, \vec{v}_n$ are linearly dependent

Ex: a) $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}3\\3\end{bmatrix}$ are linearly dependent.

$$0\begin{bmatrix}1\\0\end{bmatrix} + -3\begin{bmatrix}1\\1\end{bmatrix} + 1\begin{bmatrix}3\\3\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Alternative terminology

$\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}3\\3\end{bmatrix}\right\}$ is linearly dependent.

b) $\left\{\begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix}, \begin{bmatrix}2\\7\end{bmatrix}\right\}$ is linearly dependent.

$$-3\begin{bmatrix}0\\1\end{bmatrix} + -1\begin{bmatrix}2\\4\end{bmatrix} + 1\begin{bmatrix}2\\7\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

c) $\begin{bmatrix}0\\0\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$ are linearly dependent

$$-1\begin{bmatrix}0\\0\end{bmatrix} + 0\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

Ex: Are $\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\2\end{bmatrix}$ linearly independent?

Let $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$

$$\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \end{array}\right] \end{array}$$

$\vdots$

$c_1 = c_2 = c_3 = 0$

$\boxed{\text{Yes}}$

$c_1 = c_2 = c_3 = 0$ and other solutions

$\boxed{\text{No}}$