Week 3 Friday
January 24, 2020 9:39 AM

2.2 Solving Systems Cont'd

Ex: $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 4 & 0 \end{array}\right]$ or $\begin{cases} 1x + 2y = 0 \\ 3x + 4y = 0 \end{cases}$

Solution: $x = 0$
$y = 0$

$\boxed{\vec{x} = \vec{0}}$

Terminology

$\vec{x} = \vec{0}$ is called the trivial solution

$\left[\begin{array}{c|c} \text{anything} & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array}\right]$ is called a homogeneous system

> FACT
> A homogeneous system always has at least 1 solution: the trivial solution

Ex: Consider $m\left\{\left[\begin{array}{c|c} \underbrace{\qquad\qquad\qquad}_{n} & \begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{array}\right]\right.$ where $n > m$

How many solutions?

At least 1 solution: $\vec{x} = \vec{0}$

$\left[\begin{array}{ccccc|c} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & 1 & \end{array}\right]$ REF

≥ 1 parameter

$\boxed{\infty\text{-many solutions}}$

2.3 Span and Linear Independence

Ex: Is $\begin{bmatrix} 8 \\ -10 \end{bmatrix}$ a linear combination of $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$?

of $\begin{bmatrix}-1\\2\end{bmatrix}$ and $\begin{bmatrix}2\\-3\end{bmatrix}$?

Let $\underbrace{c_1\begin{bmatrix}-1\\2\end{bmatrix} + c_2\begin{bmatrix}2\\-3\end{bmatrix}}_{\text{lin. com.}} = \begin{bmatrix}8\\-10\end{bmatrix}$

$$\begin{bmatrix}-c_1\\2c_1\end{bmatrix} + \begin{bmatrix}2c_2\\-3c_2\end{bmatrix} = \begin{bmatrix}8\\-10\end{bmatrix}$$

$$\begin{bmatrix}-c_1+2c_2\\2c_1-3c_2\end{bmatrix} = \begin{bmatrix}8\\-10\end{bmatrix}$$

$$\begin{cases}-c_1+2c_2 = 8\\2c_1-3c_2=-10\end{cases}$$

$$\begin{array}{ccc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c}-1 & 2 & 8\\2 & -3 & -10\end{array}\right]$$

$\frac{R_1}{-1}$ $\left[\begin{array}{cc|c}1 & -2 & -8\\2 & -3 & -10\end{array}\right]$

R_2-2R_1 $\left[\begin{array}{cc|c}1 & -2 & -8\\0 & 1 & 6\end{array}\right]$

R_1+2R_2 $\begin{array}{cc} c_1 & c_2 \end{array}$ $\left[\begin{array}{cc|c}1 & 0 & 4\\0 & 1 & 6\end{array}\right]$ RREF

$c_1=4$, $c_2=6$

$\boxed{\text{YES}}$

To check: $4\begin{bmatrix}-1\\2\end{bmatrix} + 6\begin{bmatrix}2\\-3\end{bmatrix} = \begin{bmatrix}8\\-10\end{bmatrix}$ ✓

Ex: Is $\begin{bmatrix}1\\1\\2\end{bmatrix}$ a linear combination

of $\begin{bmatrix}1\\0\\1\end{bmatrix}$ and $\begin{bmatrix}1\\3\\0\end{bmatrix}$?

Let $\begin{bmatrix}1\\1\\2\end{bmatrix} = c_1\begin{bmatrix}1\\0\\1\end{bmatrix} + c_2\begin{bmatrix}1\\3\\0\end{bmatrix}$

Let $\begin{bmatrix}1\\1\\2\end{bmatrix} = \underbrace{c_1\begin{bmatrix}1\\0\\1\end{bmatrix} + c_2\begin{bmatrix}1\\3\\0\end{bmatrix}}_{\text{lin. com.}}$

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 1 & 0 & 2 \end{array}\right] \end{array}$$

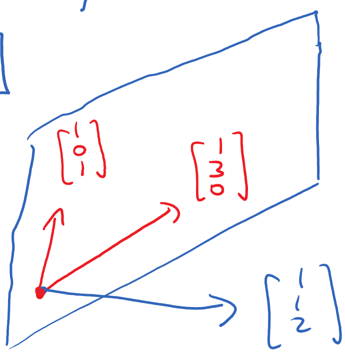
$$R_3 - R_1 \quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & -1 & 1 \end{array}\right]$$

$$\frac{R_2}{3} \quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{3} \\ 0 & -1 & 1 \end{array}\right]$$

$$R_3 + R_2 \quad \left[\begin{array}{cc|c} & & \\ & & \\ 0 & 0 & \frac{4}{3} \end{array}\right]$$

System has no solution

$\boxed{\text{No}}$



FACT

$\vec{b}$ is a linear combination of the columns of matrix A if and only if the system $[A \mid \vec{b}]$ is consistent.

(solvable)

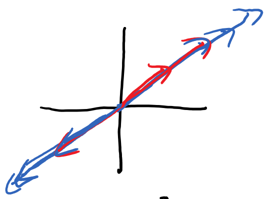
Def
The <u>span</u> of $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n$ is the set of all linear combinations of $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n$.

<u>Ex</u>: a) $\text{span}(\vec{a}, \vec{b}) = \{ \vec{0}, \vec{a}, 4\vec{a}, -\pi\vec{a}, \vec{b}, 7\vec{b}, -\sqrt{2}\,\vec{b}, 3\vec{a} - 2\vec{b}, \dots \}$

b) $\text{span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) = \{ c_1\vec{u}_1 + c_2\vec{u}_2 + \dots + c_n\vec{u}_n \}$
$c_1, c_2, \dots, c_n$: any real #

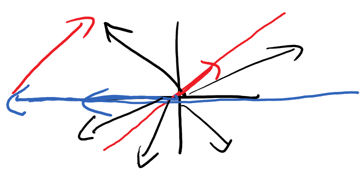
<u>Ex</u>: Describe each span geometrically

a) $\text{span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \end{bmatrix} \right)$

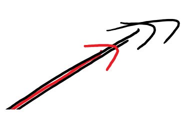


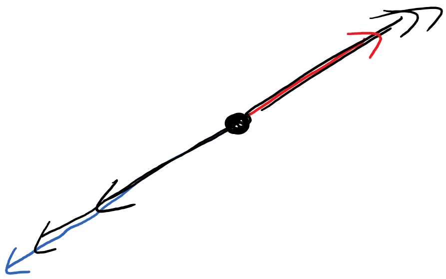
Line in $\mathbb{R}^2$ through origin

b) $\text{span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \end{bmatrix} \right)$
= all of $\mathbb{R}^2$
(entire xy-plane)

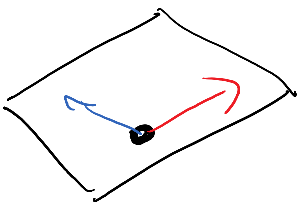


c) $\text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix} \right)$
= line in $\mathbb{R}^3$ through origin

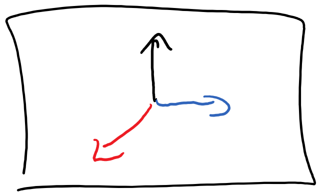




d) span$\left(\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right)$ = plane through origin in $\mathbb{R}^3$



e) span$\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)$ = all of $\mathbb{R}^3$



Fact: Every span contains $\vec{0}$ (the origin)