Week 3 Thursday

January 23, 2020 9:43 AM

2.2 Solving Systems Cont'd

Ex: Solve by Gauss-Jordan Elimination

$$\begin{array}{c} \begin{matrix} w & x & y & z & \end{matrix} \\ \left[\begin{array}{cccc|c} 1 & 1 & 2 & 10 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 1 & 3 & 4 & 12 & 9 \end{array}\right] \end{array}$$

$$R_3 - R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 2 & 10 & 5 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & 2 & 4 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ \\ R_3 - 2R_2 \end{array} \quad \begin{array}{c} \begin{matrix} w & x & y & z & \end{matrix} \\ \left[\begin{array}{cccc|c} 1 & 0 & 1 & 9 & 3 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \quad \text{RREF}$$

$\boxed{y = s}$ $\boxed{z = t}$ "parameters"

$w + y + 9z = 3 \rightarrow \boxed{w = 3 - s - 9t}$

$x + y + z = 2 \rightarrow \boxed{x = 2 - s - t}$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -9 \\ -1 \\ 0 \\ 1 \end{bmatrix} t$$

Ex: Find the intersection of the 2 lines:

$$\vec{x} = \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + s \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\vec{x} = \vec{x}$$

$$\begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + s\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$s\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} - t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}s + \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}t = \begin{bmatrix} 0 \\ -2 \\ -6 \end{bmatrix}$$

$$\begin{cases} 2s - t = 0 \\ \end{cases}$$

$$\begin{array}{ccc} s & t & \# \end{array}$$
$$\left[\begin{array}{cc|c} 2 & -1 & 0 \\ 1 & -1 & -2 \\ -1 & -1 & -6 \end{array}\right]$$

$R_1 \leftrightarrow R_2$
$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 2 & -1 & 0 \\ -1 & -1 & -6 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 + R_1$
$$\left[\begin{array}{cc|c} 1 & -1 & -2 \\ 0 & 1 & 4 \\ 0 & -2 & -8 \end{array}\right]$$

$R_1 + R_2$
$R_3 + 2R_2$
$$\begin{array}{ccc} s & t & \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} \leftarrow s = 2 \\ \leftarrow t = 4 \\ \leftarrow \text{no info} \end{array}$$

1 solution : $s = 2$, $t = 4$

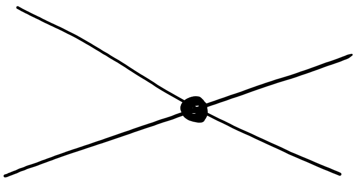
$s = 2 \rightarrow 1^{st}$ line <u>or</u> $t = 4 \rightarrow 2^{nd}$ line

$$\vec{x} = \begin{bmatrix} -5 \\ 6 \\ 5 \end{bmatrix} + \boxed{2} \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 8 \\ 3 \end{bmatrix}$$

$$\boxed{\begin{cases} x = -1 \\ y = 8 \\ z = 3 \end{cases}}$$

$(-1, 8, 3)$



Ex: How many solutions does the system have?

$$\begin{array}{cc} x & y \\ \multicolumn{2}{c}{\left[\begin{array}{cc|c} 1 & k & 1 \\ k & 1 & 1 \end{array}\right]} \end{array}$$

$R_2 - kR_1$ $\quad \left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1-k^2 & 1-k \end{array}\right]$

$1-k^2 \neq 0$

$\frac{R_2}{1-k^2}$ $\quad \left[\begin{array}{cc|c} 1 & k & 1 \\ 0 & 1 & \frac{1-k}{1-k^2} \end{array}\right]$

$\boxed{\text{1 SOLUTION}}$

$1-k^2 = 0$

$(1-k)(1+k) = 0$

$k=1$ $\quad \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$

$\boxed{\infty\text{-MANY SOLUTIONS}}$

$k=-1$ $\quad \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & 2 \end{array}\right]$

$\boxed{\text{NO SOLUTION}}$

1 solution	if	$1-k^2 \neq 0$
∞-many	if	$k=1$
0 solutions	if	$k=-1$

1 solution if $k \neq \pm 1$

Def

Rank of a matrix : # of nonzero rows in REF or RREF

Fact

When a system is consistent (has a solution):

rank + (# parameters in solution) = # variables

Ex:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & \\ 0 & 4 & 5 & \\ 0 & 0 & 0 & \end{array}\right]$$

REF ✓

rank = 2

parameters = 1

variables = 3

Rephrased: (# columns with circles) + (# columns without circles) = # columns