

2.1 Solving Systems Cont'd

$$\begin{cases} 2x - 3y = 8 \\ -4x + 6y = 20 \end{cases}$$

Augmented matrix

$$\begin{array}{cc|c} x & y & \# \\ \hline 2 & -3 & 8 \\ -4 & 6 & 20 \end{array}$$

Coefficient matrix

$$\begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix}$$

3 Elementary Row Operations

- 1) Swap 2 Rows
- 2) Multiply / Divide a Row
- 3) Current Row \pm # (Pivot Row)

Goal: Get $\begin{bmatrix} 1 & 0 & | & \\ 0 & 1 & | & \end{bmatrix}$
or as close as possible.

Ex: Solve the system above

$$\begin{array}{cc|c} x & y & \# \\ \hline 2 & -3 & 8 \\ -4 & 6 & 20 \end{array}$$

Get a 1, "the pivot"

$$\frac{R_1}{2} \quad \begin{bmatrix} 1 & -\frac{3}{2} & | & 4 \\ -4 & 6 & | & 20 \end{bmatrix}$$

Get 0's in the rest of Column 1

$$R_2 + 4R_1 \quad \begin{bmatrix} 1 & -\frac{3}{2} & | & 4 \\ 0 & 0 & | & 36 \end{bmatrix}$$

$$0x + 0y = 36 \quad \text{impossible}$$

System has no solution

FACT

If you see $[0 \ 0 \ | \ \text{nonzero}]$ or
 $[0 \ 0 \ 0 \ | \ \text{nonzero}]$
then system has no solution.

Ex: Solve

$$\begin{array}{cc|c} x & y & \# \\ \hline 2 & -3 & 8 \\ -4 & 6 & -16 \end{array}$$

Get a 1

$\frac{R_1}{2}$

$$\begin{array}{cc|c} 1 & -\frac{3}{2} & 4 \\ -4 & 6 & -16 \end{array}$$

Get 0's in Column 1

$$\begin{array}{cc|c} x & y & \# \\ \hline 1 & -\frac{3}{2} & 4 \\ 0 & 0 & 0 \end{array}$$

$R_2 + 4R_1$

Column for y has no "1"
 y is a free variable

$$\boxed{y = t} \quad (\text{any real } \#)$$

$$x - \frac{3}{2}y = 4 \rightarrow x = \frac{3}{2}y + 4 \rightarrow \boxed{x = \frac{3}{2}t + 4}$$

$$\boxed{\begin{array}{l} x = \frac{3}{2}t + 4 \\ y = t \end{array}}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} t + \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

System has infinitely-many solutions

Ex:

Solve

$$\begin{array}{c|cc} x & y & \# \\ \hline 1 & 0 & 5 \\ 2 & 3 & 4 \\ 3 & 4 & 7 \end{array}$$

Get a 1 ✓
Get 0's in Column 1

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 3 & -6 \\ 0 & 4 & -8 \end{bmatrix}$$

Get a 1

$$\frac{R_2}{3} \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 4 & -8 \end{bmatrix}$$

Get 0's in Column 2

$$\begin{array}{c} x & y & \# \\ \hline 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array} \leftarrow \text{No info}$$

$$1x + 0y = 5 \rightarrow \begin{array}{|l} x = 5 \\ y = -2 \end{array}$$

Unique Solution

Unique Solution

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & \underline{1} \end{array} \right] \text{ impossible}$$

System has no solution

Ex: Solve by back-substitution

$$\left[\begin{array}{ccc|c} x & y & z & \# \\ 4 & 1 & 1 & 15 \\ 0 & 3 & 5 & 29 \\ 0 & 0 & 2 & 8 \end{array} \right]$$

Start @ bottom

$$2z = 8 \rightarrow \boxed{z = 4}$$

$$3y + 5z = 29 \rightarrow 3y + 20 = 29 \rightarrow \boxed{y = 3}$$

$$4x + y + z = 15 \rightarrow 4x + 3 + 4 = 15 \rightarrow \boxed{x = 2}$$

Unique solution

2.2 Solving Systems

A matrix is in row-echelon form (REF) if:

1) any zero rows are at the bottom

AND

2) the leading nonzero entries of each row move down and right

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 4 & \end{array} \right]$$

REF

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & \\ 0 & 4 & -8 & \\ 0 & 0 & 0 & \end{array} \right]$$

REF

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & \\ 0 & 0 & 3 & \\ 0 & 0 & 0 & \end{array} \right]$$

REF