Week 3 Monday

January 20, 2020 7:57 AM

Test Fri Jan 31^{st}

1.1–1.3, Cross Product, 2.1–2.2

Practice Problems www.leahhoward.com

6 Questions

No Formula Sheet

(1.4) Cross Product Cont'd

Ex: $\bar{u} = [1, -2, 3]$ $\bar{v} = [-4, 5, 6]$

Find $\bar{u} \times \bar{v}$

Method I:

$$\begin{array}{ccccc} 1 & -2 & 3 & 1 & -2 \\ -4 & 5 & 6 & -4 & 5 \end{array}$$

$$\bar{u} \times \bar{v} = [-27, -18, -3]$$

Method II:

$$\bar{u} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -2 & 3 \\ -4 & 5 & 6 \end{vmatrix}$$

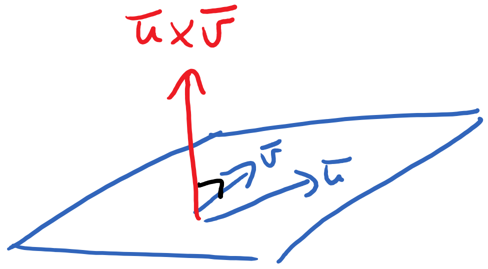
$$\begin{bmatrix} + & - & + \\ & & \end{bmatrix}$$

$$= \bar{i} \begin{vmatrix} -2 & 3 \\ 5 & 6 \end{vmatrix} - \bar{j} \begin{vmatrix} 1 & 3 \\ -4 & 6 \end{vmatrix} + \bar{k} \begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix}$$

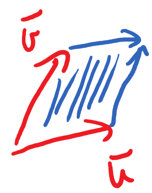
$$= \bar{i}(-27) - \bar{j}(18) + \bar{k}(-3)$$

$= [-27, -18, -3]$

Recap



$\|\bar{u} \times \bar{v}\| =$ Area of parallelogram



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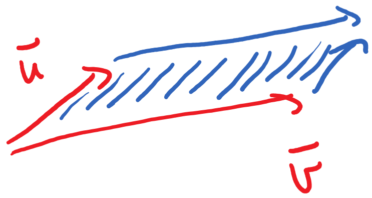
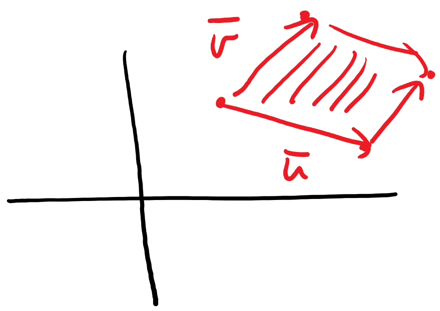
3 Geometry Formulas

1) $A(\text{parallelogram in } \mathbb{R}^3) = \|\bar{u} \times \bar{v}\|$

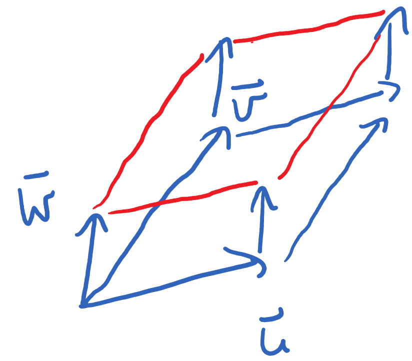


2) $A(\text{parallelogram in } \mathbb{R}^2)$

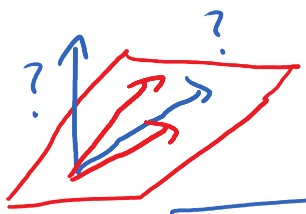
$= \text{absolute value of } \det \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}$



3) $V(\underbrace{\text{parallelepiped}}_{\text{slanted box}} \text{ in } \mathbb{R}^3) =$ absolute value of $\det \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{bmatrix}$



<u>Ex</u>: Do vectors $[1, 4, 7]$, $[2, 5, 9]$ and $[1, -2, -3]$ lie in a plane?



$$\boxed{\text{Yes if and only if } V(\text{parallelepiped}) = 0}$$

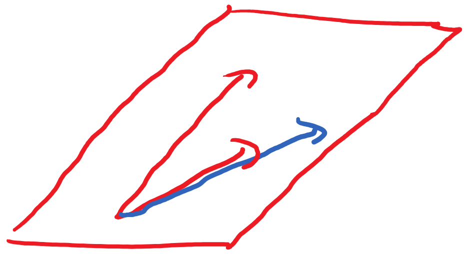
$$V(\text{parallelepiped}) = \left| \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 9 \\ 1 & -2 & -3 \end{vmatrix} \right|$$

$$= \left| 1 \begin{vmatrix} 5 & 9 \\ -2 & -3 \end{vmatrix} - 4 \begin{vmatrix} 2 & 9 \\ 1 & -3 \end{vmatrix} + 7 \begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} \right|$$

$$= \left| 1(3) - 4(-15) + 7(-9) \right|$$

$= |0|$

$= 0$ $\boxed{\text{Yes}}$



Ex: Area of parallelogram determined by $[1,6]$ and $[3,5]$?

$$\text{area} = \left| \begin{vmatrix} 1 & 6 \\ 3 & 5 \end{vmatrix} \right|$$

$$= |-13|$$

$$= 13$$

2.1 Linear Systems

Linear equation:

$$ax + by + cz = d$$

a,b,c,d : real #

Linear system:

$$\begin{cases} 2x - y + 3z = 1 \\ \quad 4y - 2z = 8 \end{cases}$$

FACT

A system can have:

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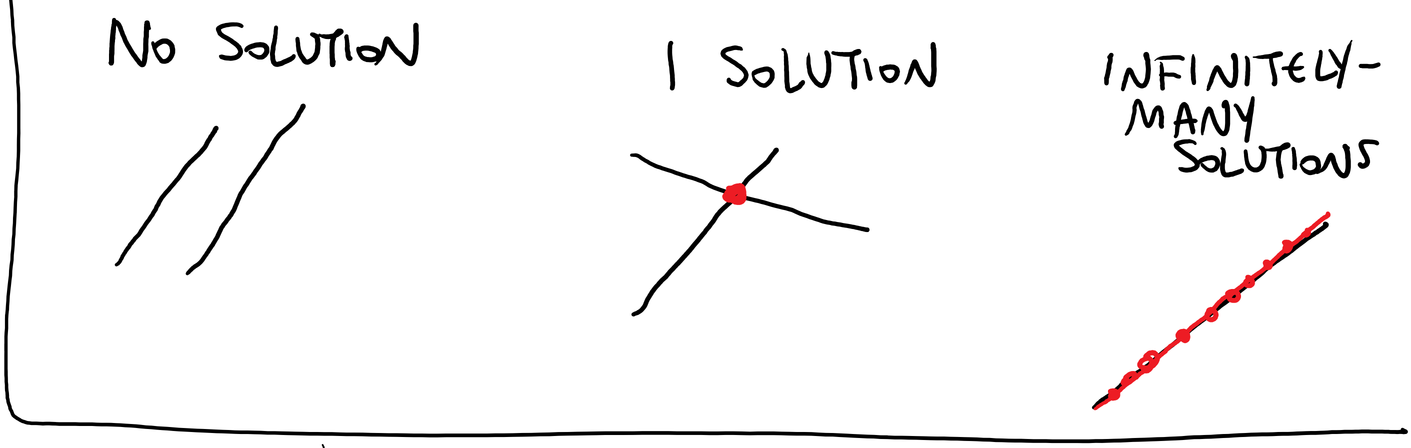

"inconsistent system"

"consistent system" (solvable system)

<u>Ex</u>: Solve by elimination

$$\begin{cases} 2x + 6y = -14 \\ -3x + 3y = -15 \end{cases}$$

$$\begin{array}{c} \begin{array}{ccc} x & y & \# \end{array} \\ \left[\begin{array}{cc|c} 2 & 6 & -14 \\ -3 & 3 & -15 \end{array}\right] \end{array}$$

Get a 1

$R_1/2$ $\left[\begin{array}{cc|c} 1 & 3 & -7 \\ -3 & 3 & -15 \end{array}\right]$

Get 0's in rest of Column 1

$R_2 + 3R_1$ $\left[\begin{array}{cc|c} 1 & 3 & -7 \\ 0 & 12 & -36 \end{array}\right]$

Get a 1 Goal: $\left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \end{array}\right]$

$R_2/12$ $\left[\begin{array}{cc|c} 1 & 3 & -7 \\ 0 & 1 & -3 \end{array}\right]$

Get 0's in rest of Column 2

$R_1 - 3R_2$ $\begin{array}{c} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \end{array}\right] \\ \begin{array}{ccc} x & y & \end{array} \end{array}$

$1x + 0y = 2 \rightarrow \boxed{\begin{array}{l} x = 2 \\ y = -3 \end{array}}$