Week 2 Friday
January 17, 2020 9:35 AM

# Cross Product Cont'd

**Ex:** $\vec{u} = [1,2,3]$ and $\vec{v} = [4,5,6]$

Find $\vec{u} \times \vec{v}$

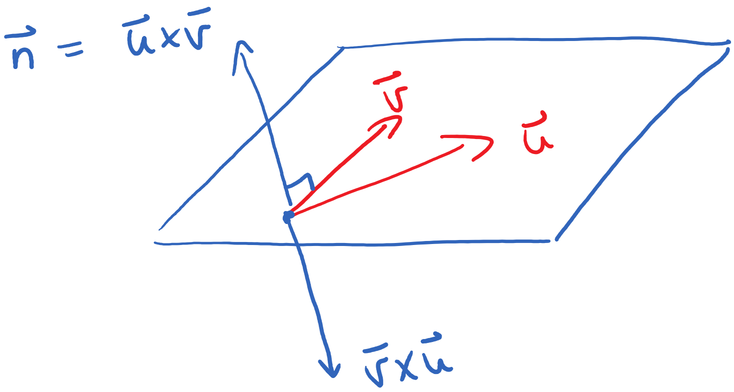
$$\begin{array}{ccccc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \end{array}$$

$\vec{u} \times \vec{v} = [2(6) - 3(5), \ \ldots \ ]$

$= [-3, 6, -3]$

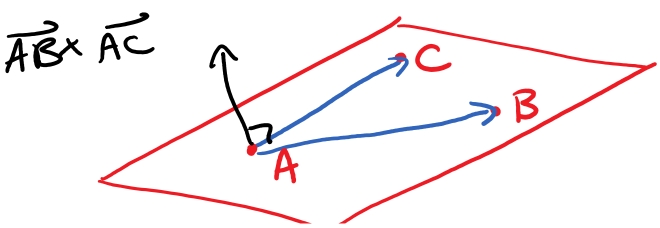
**Facts**

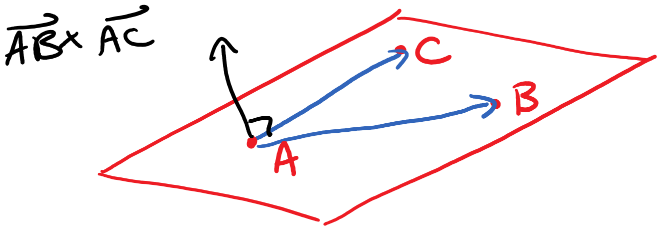
1) $\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$
2) $\vec{u} \times \vec{v}$ is $\perp$ to $\vec{u}$ and $\vec{v}$




Right Hand Rule

**Ex:** Find the general form of the plane through $A = (1,3,6)$, $B = (2,1,4)$ and $C = (1,-1,5)$






$\vec{AB} = [1,-2,-2]$

$\vec{AC} = [0,-4,-1]$

$\vec{n} = \vec{AB}\times\vec{AC} = [-6,1,-4]$

$$\boxed{\begin{array}{cccccc} 1 & -2 & -2 & 1 & -2 \\ 0 & -4 & -1 & 0 & -4 \end{array}}$$

Normal form $\quad \vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$

$$\begin{bmatrix}-6\\1\\-4\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-6\\1\\-4\end{bmatrix}\cdot\begin{bmatrix}1\\3\\6\end{bmatrix}$$

General form $\quad -6x + y - 4z = -27$

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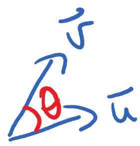
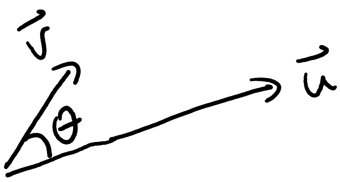
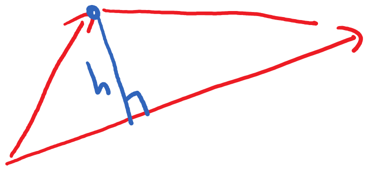
Recall $\quad \vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$




$$\boxed{\text{FACT} \quad \|\vec{u}\times\vec{v}\| = \|\vec{u}\|\,\|\vec{v}\|\sin\theta}$$

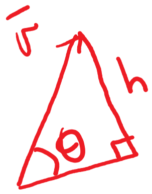
<u>Ex</u>: Show that the triangle below has area $= \frac{1}{2}\|\vec{u}\times\vec{v}\|$ :





$A = \frac{1}{2} b h$

$= \frac{1}{2} \|\bar{u}\| h$



$\sin\theta = \dfrac{h}{\|\bar{v}\|}$

$h = \|\bar{v}\| \sin\theta$

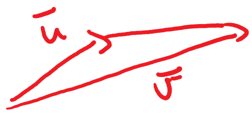
$A = \frac{1}{2} \|\bar{u}\| \, \|\bar{v}\| \sin\theta$

$= \frac{1}{2} \| \bar{u} \times \bar{v} \|$

FACT

For $\vec{u}$ and $\bar{v}$ in $\mathbb{R}^3$:

Area (triangle) $= \frac{1}{2} \| \bar{u} \times \bar{v} \|$



Area (parallelogram) $= \| \bar{u} \times \bar{v} \|$

Ex: Find the area of the triangle determined by $\bar{u} = [1, 4, 5]$ and $\bar{v} = [2, 3, 6]$.


$$\boxed{\begin{array}{ccccc} 1 & 4 & 5 & 1 & 4 \\ 2 & 3 & 6 & 2 & 3 \end{array}}$$

$$\vec{u} \times \vec{v} = [9, 4, -5]$$

$$\| \vec{u} \times \vec{v} \| = \sqrt{81 + 16 + 25} = \sqrt{122}$$

$$A(\Delta) = \frac{\sqrt{122}}{2}$$

**Def**

<u>Matrix</u> : rectangular array

e.g. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

**Def**

<u>Size</u> of a matrix : (#rows) x (#columns)

e.g. $A$ is $2 \times 3$

**NOTATION**

The <u>determinant</u> of $A$ is written

$\det A$ or $|A|$

## FORMULAS

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{matrix} a & & b \\ & \times & \\ c & & d \end{matrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

"Cofactor Expansion"

Signs alternate $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

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Ex: Calculate $\det\begin{bmatrix} 1 & 4 & 6 \\ 2 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$

$$= \begin{vmatrix} 1 & 4 & 6 \\ 2 & 1 & 3 \\ 0 & 6 & 7 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 0 & 7 \end{vmatrix} + 6\begin{vmatrix} 2 & 1 \\ 0 & 6 \end{vmatrix}$$

$$= 1(-11) - 4(14) + 6(12)$$

$$= 5$$

$$\vec{i} = [1,0,0] \qquad \vec{j} = [0,1,0] \qquad \vec{k} = [0,0,1]$$

## 2nd Method for Cross Product

$$[2,1,3] \times [-6,4,2] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 3 \\ -6 & 4 & 2 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 3 \\ -6 & 2 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 1 \\ -6 & 4 \end{vmatrix}$$

$$= \vec{i}(-10) - \vec{j}(22) + \vec{k}(14)$$

$$= [-10,0,0] + [0,-22,0] + [0,0,14]$$

$$= [-10, -22, 14]$$