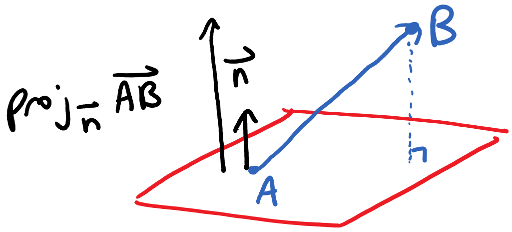
Week 2 Tuesday

January 14, 2020 9:10 AM

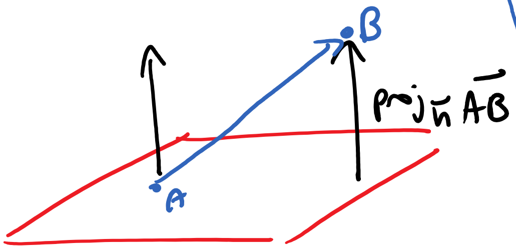
1.3 Lines and Planes Cont'd

Ex: Find the distance between $B=(1,3,3)$ and the plane $x+y+2z=7$.



Choose any point on plane

$A=(1,2,2)$



$$\text{distance} = \|\text{proj}_{\vec{n}}\overrightarrow{AB}\|$$

$$\overrightarrow{AB} = \begin{bmatrix}0\\1\\1\end{bmatrix} \quad \text{or} \quad [0,1,1]$$

$$\vec{n} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$\text{proj}_{\vec{n}}\overrightarrow{AB} = \frac{\vec{n}\cdot\overrightarrow{AB}}{\|\vec{n}\|^2}\vec{n}$$

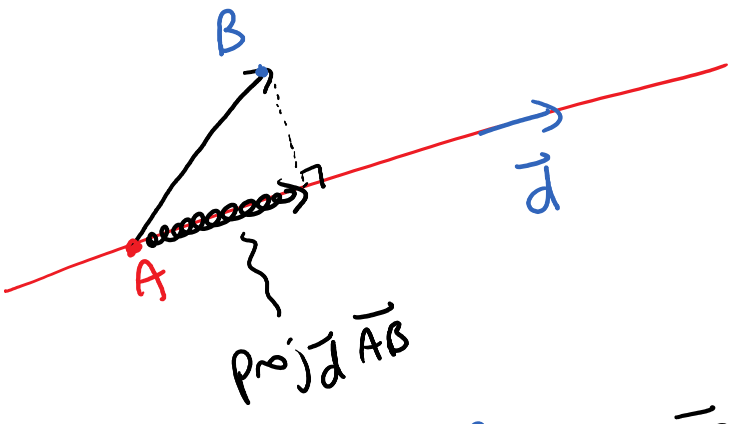
$$= \frac{3}{6}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

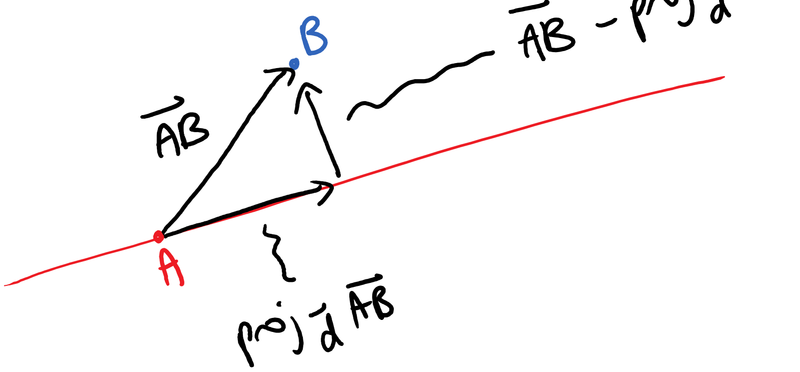
$$= \frac{1}{2}\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$$\text{distance} = \left\|\frac{1}{2}\begin{bmatrix}1\\1\\2\end{bmatrix}\right\| = \frac{\sqrt{6}}{2}$$

Recall $\boxed{\|c\vec{v}\| = |c|\,\|\vec{v}\|}$

<u>Ex</u>: Find the distance between $B = (1,1,0)$ and the line through $A = (0,1,2)$ with $\vec{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.





$$\text{distance} = \left\| \vec{AB} - \text{proj}_{\vec{d}}\,\vec{AB} \right\|$$

$A = (0,1,2)$ $\quad B = (1,1,0)$ $\quad \vec{d} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\vec{AB} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\text{proj}_{\vec{d}}\,\vec{AB} = \frac{\vec{d} \cdot \vec{AB}}{\|\vec{d}\|^2}\,\vec{d}$$

$$= \frac{-1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

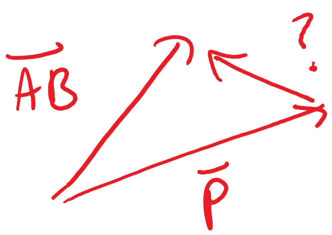
$$\overrightarrow{AB} - \text{proj}_{\vec{d}}\,\overrightarrow{AB} = \begin{bmatrix}1\\0\\-2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}\frac{3}{2}\\0\\-\frac{3}{2}\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}3\\0\\-3\end{bmatrix}$$

$$\text{distance} = \left\| \frac{1}{2}\begin{bmatrix}3\\0\\-3\end{bmatrix} \right\|$$

$$= \frac{\sqrt{18}}{2} \quad \text{or} \quad \frac{3\sqrt{2}}{2}$$



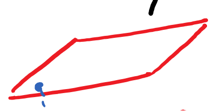
$$\vec{p} + ? = \overrightarrow{AB}$$

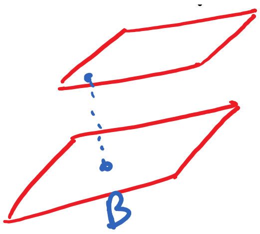
$$? = \overrightarrow{AB} - \vec{p}$$

$$= \overrightarrow{AB} - \text{proj}_{\vec{d}}\,\overrightarrow{AB}$$

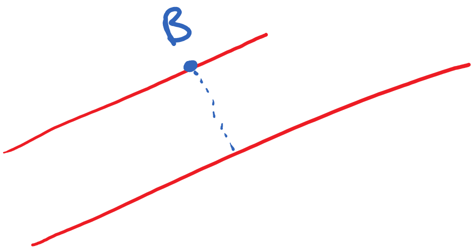
Comments

1) For distance between parallel planes, choose any point on either plane as B.

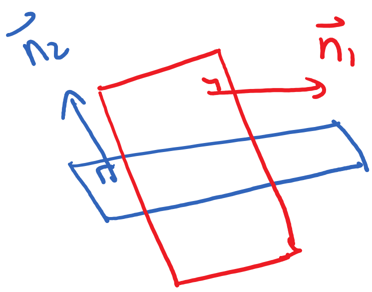


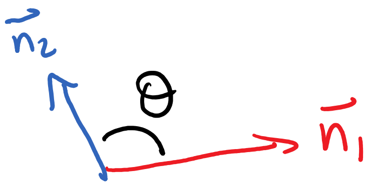


2) Similarly for parallel lines



Def
The <u>angle between planes</u> is the angle between their normals.





Parallel planes have parallel normals
Perpendicular " perpendicular "

$$2x - 4y + z = 9 \qquad \bar{n} = [2, -4, 1]$$

(1.4) Cross Product

Cross Product $\bar{u} \times \bar{v}$ is defined for $\bar{u}, \bar{v}$ in $\mathbb{R}^3$

Ex: $\bar{u} = [1, 2, 1]$ $\bar{v} = [3, -1, 4]$

$$\begin{array}{cccccc} 1 & 2 & 1 & 1 & 2 \\ 3 & -1 & 4 & 3 & -1 \end{array}$$

$$\bar{u} \times \bar{v} = [2(4) - 1(-1), 1(3) - 1(4), 1(-1) - 2(3)]$$

$$= [9, -1, -7]$$

Ex: a) $\bar{v} \times \bar{u}$

$$\begin{array}{cccccc} 3 & -1 & 4 & 3 & -1 \\ 1 & 2 & 1 & 1 & 2 \end{array}$$

$$\bar{v} \times \bar{u} = [-9, 1, 7]$$

b) $(\bar{u} \times \bar{v}) \cdot \bar{u}$

$$= [9, -1, -7] \cdot [1, 2, 1]$$

$$= 0$$

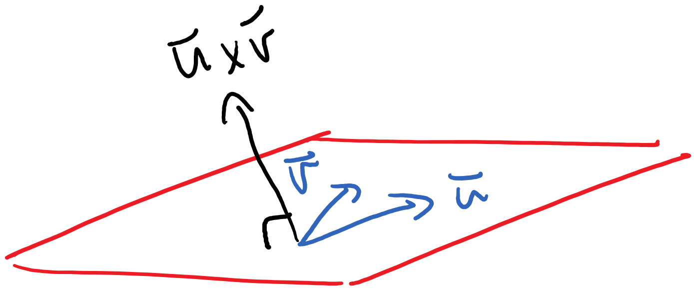
FACTS

1) $\bar{v} \times \bar{u} = -(\bar{u} \times \bar{v})$

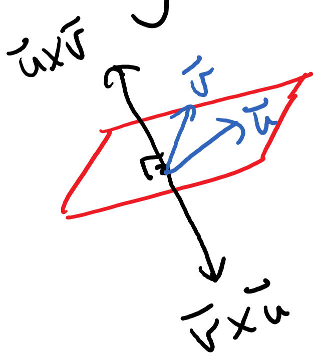
2) $\bar{u} \times \bar{v}$ is perpendicular to $\bar{u}$ and $\bar{v}$ for all $\bar{u}, \bar{v}$ in $\mathbb{R}^3$

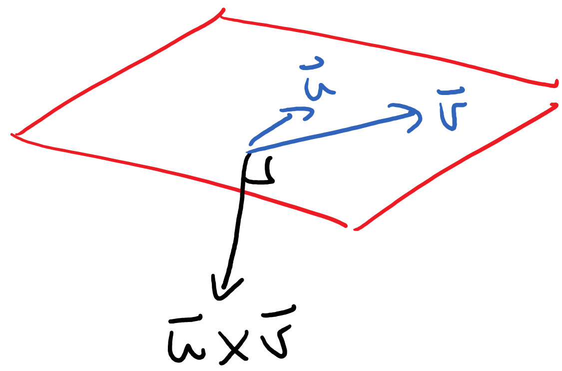
FACT

$\bar{u} \times \bar{v}$ is a normal for the plane containing $\bar{u}$ and $\bar{v}$



Direction of $\bar{u} \times \bar{v}$ comes from Right Hand Rule




$\bar{u}$
$\bar{v}$
$\bar{u} \times \bar{v}$