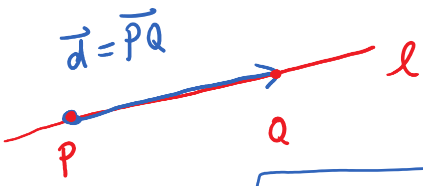
Week 2 Monday

January 13, 2020 7:56 AM

1.3 Lines and Planes Cont'd

Lines in $\mathbb{R}^3$

Ex: Vector and parametric form of the line in $\mathbb{R}^3$ through $P(2,1,12)$ and $Q(0,-3,6)$?



$\vec{d}=\vec{PQ}=\begin{bmatrix}-2\\-4\\-6\end{bmatrix}$

Vector Form $\quad \vec{x}=\vec{p}+t\vec{d}$

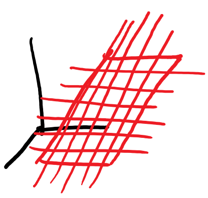
$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\12\end{bmatrix}+t\begin{bmatrix}-2\\-4\\-6\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\1\\12\end{bmatrix}+\begin{bmatrix}-2t\\-4t\\-6t\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2-2t\\1-4t\\12-6t\end{bmatrix}$$

Parametric Form $\quad \begin{cases}x=2-2t\\y=1-4t\\z=12-6t\end{cases}$

FACT $ax+by+cz=d$ is the general form of a plane in $\mathbb{R}^3$

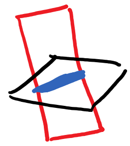


Infinite flat surface

Note: General/normal form of a line in $\mathbb{R}^3$

requires 2 equations:

$$\begin{cases} x + 2y + 3z = 6 \\ 2x - y + z = 1 \end{cases}$$



Don't use general/normal form for lines in $\mathbb{R}^3$ (inconvenient)

Planes in $\mathbb{R}^3$

Ex: Normal and general form of plane in $\mathbb{R}^3$ with normal $\begin{bmatrix}1\\1\\2\end{bmatrix}$ through point $P(1,-1,3)$?

Normal form $\quad \vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$

$$\begin{bmatrix}1\\1\\2\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\1\\2\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\3\end{bmatrix}$$

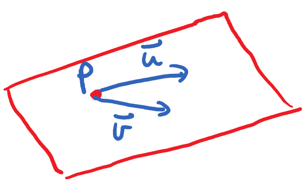
General form $\quad 1x + 1y + 2z = 6$

Def

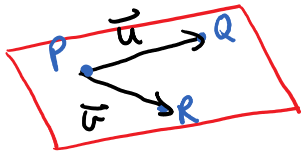
Vector form of a plane in $\mathbb{R}^3$ is

$$\vec{x} = \vec{p} + s\vec{u} + t\vec{v} \quad \text{where}$$

$\vec{u}, \vec{v}$ are nonparallel direction vectors and s, t are any real numbers



Ex: Vector and parametric form of plane through $P(6,0,0)$, $Q(0,6,0)$ and $R(0,0,3)$?



direction vectors $\quad \vec{u} = \vec{PQ} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$ Think Q-P

$\vec{v} = \vec{PR} = \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$

not parallel
(not multiples of one another)

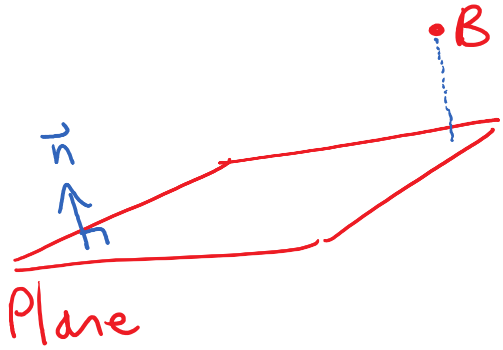
Vector Form $\quad \vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

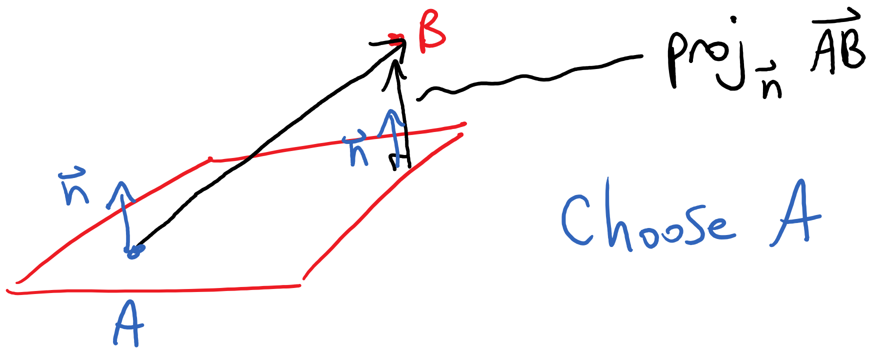
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix} + t \begin{bmatrix} -6 \\ 0 \\ 3 \end{bmatrix}$$

Parametric Form
$$\begin{cases} x = 6 - 6s - 6t \\ y = 6s \\ z = 3t \end{cases}$$

	Lines in $\mathbb{R}^2$	Lines in $\mathbb{R}^3$	Planes in $\mathbb{R}^3$
Vector	$\vec{x} = \vec{p} + t\vec{d}$	$\vec{x} = \vec{p} + t\vec{d}$	$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$
Parametric	$\begin{cases} x = \\ y = \end{cases}$	$\begin{cases} x = \\ y = \\ z = \end{cases}$	$\begin{cases} x = \\ y = \\ z = \end{cases}$
General	$ax + by = c$	No	$ax + by + cz = d$
Normal	$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$	No	$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

Ex: Find the distance between $B = (1,3,3)$ and $x + y + 2z = 7$.





Choose A

$$\text{distance} = \| \text{proj}_{\vec{n}} \overrightarrow{AB} \|$$