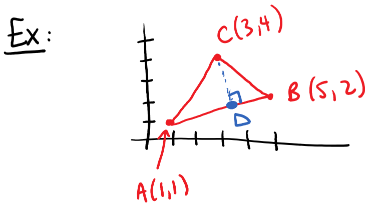
Week 1 Friday

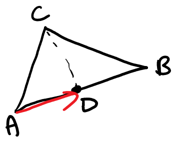
January 10, 2020 9:39 AM

1.2 Cont'd

Ex:



Find D



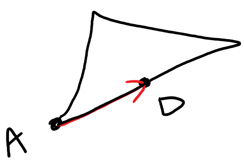
$\vec{AD} = \text{proj}_{\vec{AB}} \vec{AC}$ "projection of $\vec{AC}$ onto $\vec{AB}$"

$$\boxed{\begin{array}{l} \vec{AB} = [4,1] \\ \vec{AC} = [2,3] \end{array}}$$

$$= \frac{\vec{AB} \cdot \vec{AC}}{\|\vec{AB}\|^2} \vec{AB}$$

$$= \frac{11}{17}[4,1]$$

$$= \frac{1}{17}[44,11]$$



$\vec{A} + \vec{AD} = \vec{D}$ Formally $\vec{A} = [1,1]$

$$\vec{D} = \frac{17}{17}[1,1] + \frac{1}{17}[44,11]$$

$$= \frac{1}{17}[17,17] + \frac{1}{17}[44,11]$$

$$= \frac{1}{17}[61,28]$$

$$D = \left(\frac{61}{17}, \frac{28}{17}\right) \approx (3.6, 1.6)$$

1.3 Lines and Planes

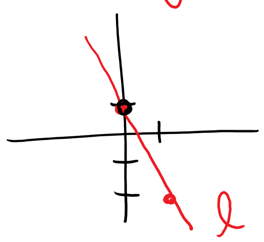
Lines in $\mathbb{R}^2$

Def
General form of a line in $\mathbb{R}^2$ is $ax+by=c$

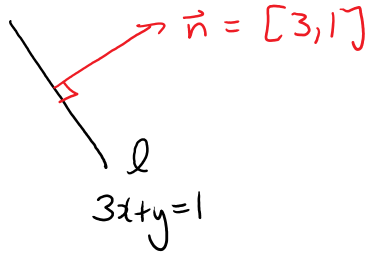
Ex: Line ℓ: $3x+y=1$

$x=0 \to y=1$ $P\ (0,1)$

$x=1 \to y=-2$ $Q\ (1,-2)$



Def
The normal vector $\vec{n}$ is orthogonal to ℓ
Its components are the coefficients of general form.



Def
The normal form of a line in $\mathbb{R}^2$ is

$$\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p} \quad \text{where}$$

$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

$\vec{p}$ = vectorization of any point on the line

Ex: Same line $3x+y=1$

$\vec{n} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$

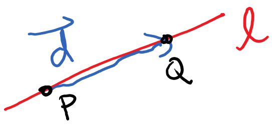
$P = (0,1) \qquad \vec{p} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

NORMAL FORM $\qquad \vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

GENERAL FORM $\qquad 3x + y = 1$

Def
A <u>direction vector</u> for a line is
$\vec{d} = \overrightarrow{PQ}$, where P and Q are points on the line



Def
The <u>vector form</u> of a line in $\mathbb{R}^2$ is:

$$\vec{x} = \vec{p} + t\vec{d}$$

$\vec{p}$ = vectorization of any point on line
t = any real #

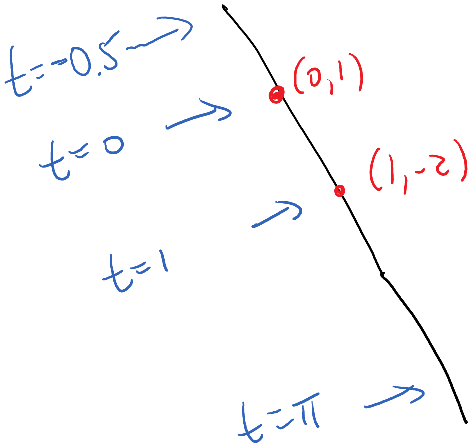
<u>Ex</u>: Same line $\quad 3x + y = 1$

$P = (0,1) \qquad Q = (1,-2)$

$$\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Vector Form $\quad \vec{x} = \vec{p} + t\,\vec{d}$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} + t\begin{bmatrix}1\\-3\end{bmatrix}$$



$\boxed{\begin{array}{l}\text{Def} \\ \text{Parametric form of a line in } \mathbb{R}^2: \\ \quad \begin{cases} x = \\ y = \end{cases}\end{array}}$

Vector Form

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} + t\begin{bmatrix}1\\-3\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}t\\-3t\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}t\\1-3t\end{bmatrix}$$

Parametric Form

$$\begin{cases} x = t \\ y = 1-3t \end{cases}$$

SUMMARY Four forms for a line in $\mathbb{R}^2$

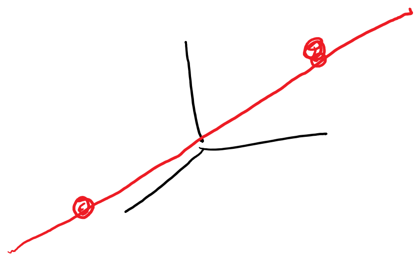
General $\qquad 3x + y = 1$

Normal $\qquad \begin{bmatrix}3\\1\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\end{bmatrix}$

Vector $\qquad \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} + t\begin{bmatrix}1\\-3\end{bmatrix}$

$\vec{d} = \vec{PQ}$

Parametric $\qquad \begin{cases} x = t \\ y = 1 - 3t \end{cases}$



Vector form will work well for lines in 3D