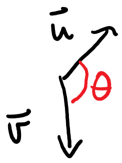
Week 1 Thursday

January 9, 2020 9:38 AM

1.2 Length and Angle Cnt'd

Fact

The angle θ between $\vec{u}$ and $\vec{v}$ is defined to be $0^\circ \le \theta \le 180^\circ$



FORMULA

$$\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

for any $\vec{u}, \vec{v}$ in $\mathbb{R}^n$

Comments

1) In $\mathbb{R}^4$ and higher dimensions, this is a definition of θ

2) θ is always defined

Ex: Find the angle between $\vec{u} = [1,-4]$ and $\vec{v} = [2,3]$

$$\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

$$-10 = \sqrt{17}\,\sqrt{13}\,\cos\theta$$

$\boxed{1(2)+(-4)(3)}$

$\boxed{\sqrt{2^2+3^2}}$

$$\frac{-10}{\sqrt{17}\,\sqrt{13}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-10}{\sqrt{17}\,\sqrt{13}}\right)$$

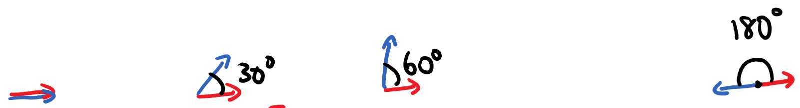
$$\approx 132^\circ$$

The sign of $\vec{u}\cdot\vec{v}$ gives some info about θ :

The sign of $\vec{u}\cdot\vec{v}$ gives some info about θ:

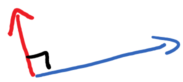
$\vec{u}\cdot\vec{v} > 0$	$\vec{u}\cdot\vec{v} = 0$	$\vec{u}\cdot\vec{v} < 0$
$\cos\theta > 0$	$\cos\theta = 0$	$\cos\theta < 0$
$0^\circ \le \theta < 90^\circ$	$\theta = 90^\circ$	$90^\circ < \theta \le 180^\circ$

Suppose $\vec{u}$ and $\vec{v}$ are unit vectors.
Then $\vec{u}\cdot\vec{v}$ gives the value of $\cos\theta$



$\vec{u}\cdot\vec{v} = 1$ $\quad\vec{u}\cdot\vec{v} = \frac{\sqrt{3}}{2}$ $\quad\vec{u}\cdot\vec{v} = \frac{1}{2}$ $\quad\vec{u}\cdot\vec{v} = -1$

Def
$\vec{u}$ and $\vec{v}$ are <u>orthogonal</u> if $\vec{u}\cdot\vec{v} = 0$



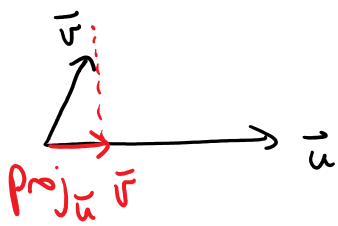
$\vec{u}$ and $\vec{v}$ are perpendicular (geometry)
" orthogonal (algebra)

The <u>projection</u> of $\vec{v}$ onto $\vec{u}$ is
written $\text{proj}_{\vec{u}}\vec{v}$

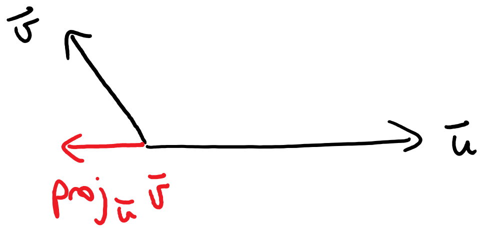
Could be read as "projection onto $\vec{u}$ of $\vec{v}$"

<u>Quick Ex:</u>

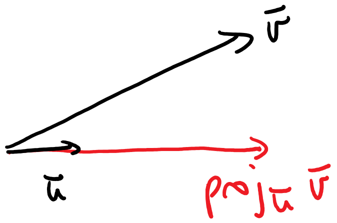
a)



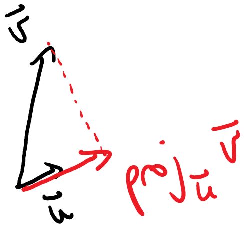
b)



c)



d)

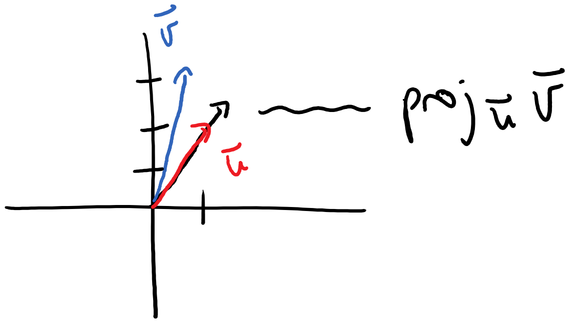


$$\boxed{\text{proj}_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u}}$$

Ex: Find $\text{proj}_{\vec{u}}\vec{v}$ for $\vec{u} = [1, 2]$ and $\vec{v} = [1, 3]$

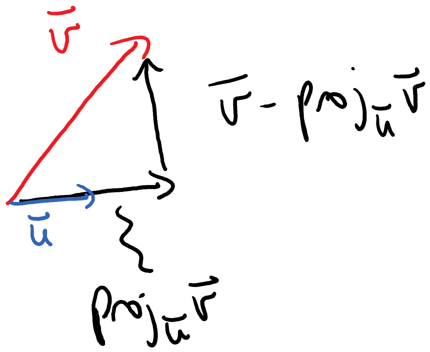
$$\text{proj}_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u}$$

$$= \frac{7}{5}[1, 2]$$



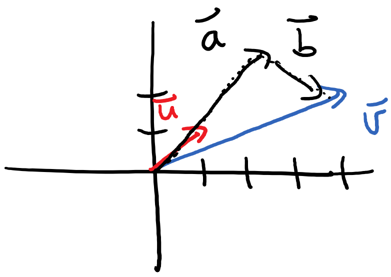
FACT

Given $\vec{u}$ and $\vec{v}$, we can decompose $\vec{v}$ into vectors parallel and perpendicular to $\vec{u}$.



Ex: $\vec{u} = [1,1]$ $\quad \vec{v} = [4,2]$

Find $\vec{a}$ and $\vec{b}$ so that $\vec{v} = \vec{a} + \vec{b}$, $\vec{a}$ is parallel to $\vec{u}$ and $\vec{b}$ is perpendicular to $\vec{u}$.



$$\vec{a} = \text{proj}_{\vec{u}} \vec{v}$$

$$= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \vec{u}$$

$$= \frac{6}{2}[1,1]$$

$$= 3[1,1]$$

$$= [3,3]$$

$$\vec{a} + \vec{b} = \vec{r}$$

$$\vec{b} = \vec{r} - \vec{a}$$

$$= [4,2] - [3,3]$$

$$= [1,-1]$$

No formula sheet for the course.