Week 1 Wednesday

January 8, 2020 8:43 AM

Coursepack : D2L or
www.leahhoward.com/251CP.pdf

Can print at Satellite Printshop

Do Sugg HW Section 1.1

Note: Coursepack uses bold font for vectors
c : scalar
$\mathbf{c}$: vector

1.2 Length and Angle

$\vec{u} = [1, 4, 2, -9]$

$\vec{v} = [2, 3, -2, -1]$

dot product $\vec{u} \cdot \vec{v} = 1(2) + 4(3) + 2(-2) + (-9)(-1)$

$= 19$

Ex: a) $[1, 5] \cdot [2, -3]$

$= 1(2) + 5(-3)$

$= -13$

b) $[1, 5] \cdot [2, -3, 0]$

undefined

c) $[u_1, u_2] \cdot [u_1, u_2]$

$= u_1^2 + u_2^2$

FACT

3 Properties about Dot Product

1) $\vec{u} \cdot \vec{u} \geq 0$

2) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$ for all $\vec{u}, \vec{v}$

3) $\vec{u} \cdot \vec{u} = 0$ if and only if $\vec{u} = \vec{0}$

Means:

3a) If $\vec{u} \cdot \vec{u} = 0$ then $\vec{u} = \vec{0}$ LESS OBVIOUS

3b) If $\vec{u} = \vec{0}$ then $\vec{u} \cdot \vec{u} = 0$ OBVIOUS

EX: Simplify

a) $(\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$

$= \vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$

$= \vec{u} \cdot \vec{u} + 2\,\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{v}$

b) $3\vec{u} \cdot (-2\vec{v} + 7\vec{w})$

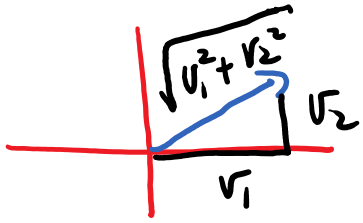
$= -6\,\vec{u}\cdot\vec{v} + 21\,\vec{u}\cdot\vec{w}$

Caution: $\vec{u}\,\vec{v}$ is meaningless

Def

The length of $\vec{v} = [v_1, v_2, \ldots, v_n]$

is $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$



Ex: a) $\|[1, 1, 1, -2]\|$

$= \sqrt{1+1+1+4}$

$= \sqrt{7}$

b) $\|[3, -1]\|$

$= \sqrt{9+1}$

$= \sqrt{10}$

Notice $[3,-1] \cdot [3,-1] = 9+1 = 10$

FACT $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$ for any $\vec{u}$

Ex: Let $\vec{v} = [v_1, v_2, v_3]$

Simplify $\|-3\vec{v}\|$

$$= \|[-3v_1, -3v_2, -3v_3]\|$$

$$= \sqrt{9v_1^2 + 9v_2^2 + 9v_3^2}$$

$$= \sqrt{9(v_1^2 + v_2^2 + v_3^2)}$$

$$= 3\sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$= 3\|\vec{v}\|$$

FACT

$$\|c\vec{a}\| = |c|\,\|\vec{a}\|$$

Def

A unit vector has length 1.

To normalize a vector means find a unit vector in the same direction.

FACT

$\vec{u} = \frac{1}{\|\vec{v}\|}\vec{v}$ has length 1 and points in the same direction as $\vec{v}$

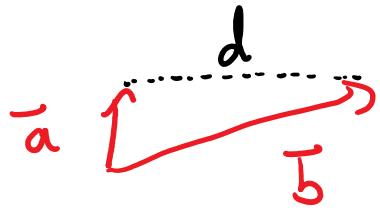
Ex: Normalize $\vec{v} = [4, -2, 1]$

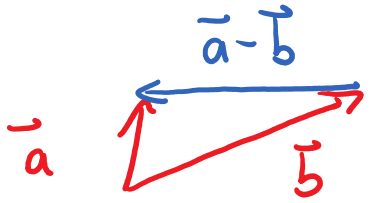
$$\frac{1}{\sqrt{21}}[4, -2, 1]$$

FACT

The distance between $\vec{a}$ and $\vec{b}$ is

$$d(\vec{a}, \vec{b}) = \|\vec{a} - \vec{b}\|$$





Ex: Find distance between $[3,7]$ and $[-1,2]$

$$d = \| [3,7] - [-1,2] \|$$

$$= \| [4,5] \|$$

$$= \sqrt{41}$$

FACT: Triangle Inequality

$\| \vec{u} + \vec{v} \| \leq \| \vec{u} \| + \| \vec{v} \|$ for any $\vec{u}, \vec{v}$