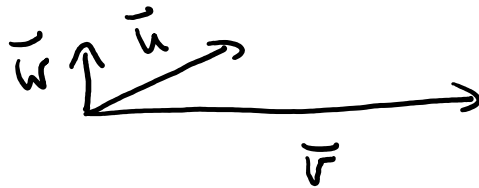


Cowsepac

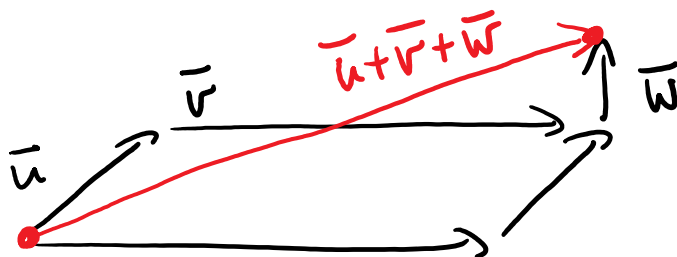
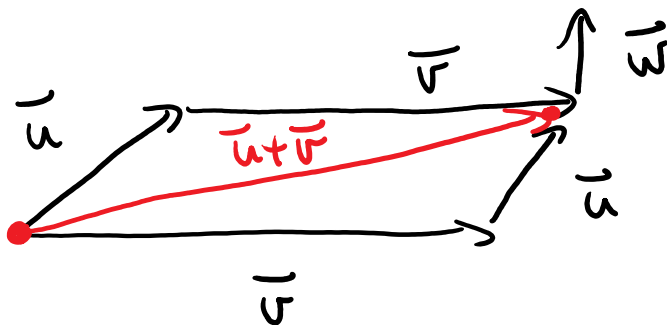
D2L or www.teahoward.com/251CP.pdf

1.1 Geometry and Algebra of Vectors Cont'd

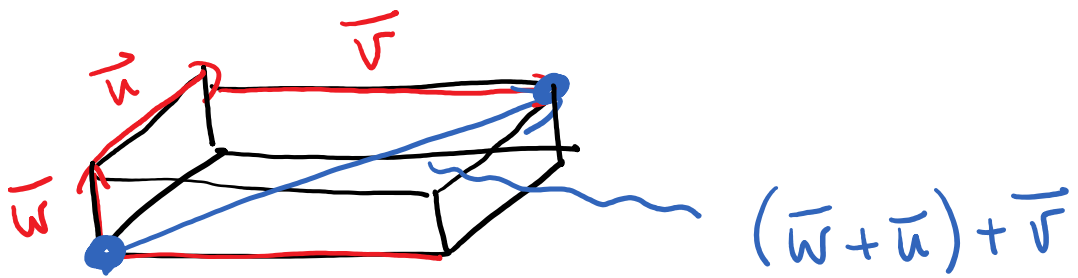
Ex:



Draw $\vec{u} + \vec{v} + \vec{w}$



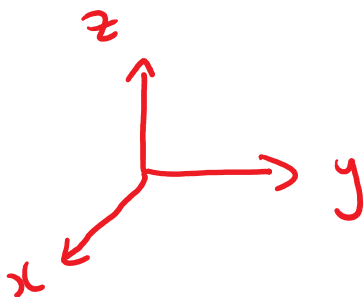
Order doesn't matter when adding vectors

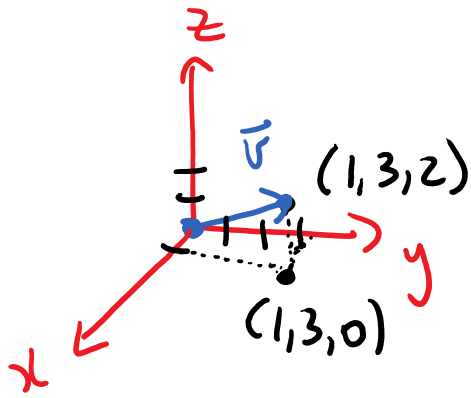


$$\begin{aligned}
 & (\vec{u} + \vec{v}) + \vec{w} \\
 &= (\vec{w} + \vec{u}) + \vec{v} \\
 & \text{etc.}
 \end{aligned}$$

Notation: \vec{v} in \mathbb{R}^n means
 \vec{v} has n components and each
 component is a real #

Ex: $\vec{v} = [1, 3, 2]$
 \vec{v} is in \mathbb{R}^3
 Draw \vec{v}





Terminology: A vector that starts at the origin is in standard position.

Zero vector $\vec{0}$

$$\text{In } \mathbb{R}^2, \quad \vec{0} = [0, 0]$$

$$\text{In } \mathbb{R}^3, \quad \vec{0} = [0, 0, 0]$$

etc.

Useful for algebra

Ex: Let \vec{u} be in \mathbb{R}^2

$$\text{Show that } \vec{u} + (-\vec{u}) = \vec{0}$$

Algebra

$$\vec{u} = [u_1, u_2]$$

Start with more complicated side

$$\begin{aligned}\vec{u} + (-\vec{u}) &= [u_1, u_2] + [-u_1, -u_2] \\ &= [0, 0] \\ &= \vec{0} \quad \checkmark\end{aligned}$$

Ex: Solve for \vec{x}

given $7\vec{x} - \vec{a} = 3(\vec{a} + 4\vec{x})$

Usual arithmetic rules apply

$$7\vec{x} - \vec{a} = 3\vec{a} + 12\vec{x}$$

$$-5\vec{x} = 4\vec{a}$$

$$\vec{x} = -\frac{4}{5}\vec{a}$$

Ex: $\vec{w} = -3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

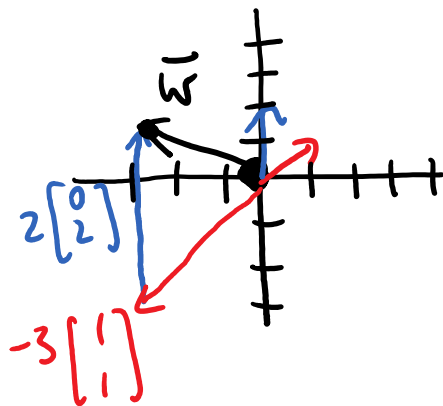
Terminology: \vec{w} is a linear combination
of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$,

with coefficients -3 and 2.

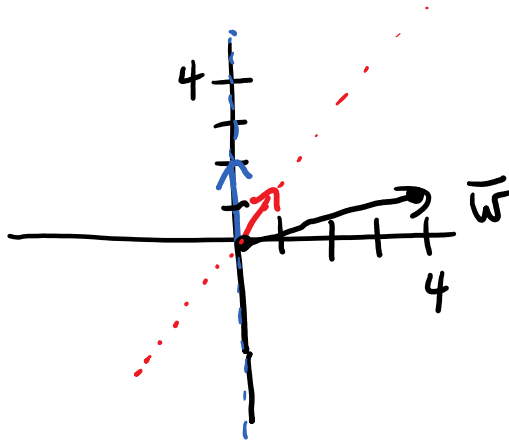
a) Find \bar{w} algebraically

$$\begin{aligned}\bar{w} &= \begin{bmatrix} -3 \\ -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} -3 \\ 1 \end{bmatrix}\end{aligned}$$

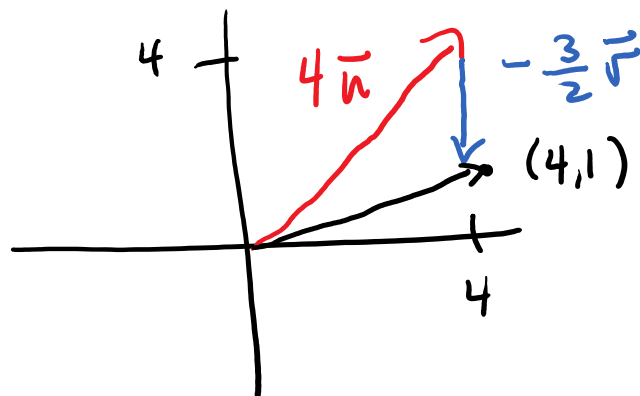
b) Find \bar{w} geometrically



Ex: Write $\bar{w} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ as a linear combination of $\bar{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ by graphing.



Think of \bar{u} and \bar{v} as the axes.



$$\begin{aligned} k(2) &= -3 \\ k &= -\frac{3}{2} \end{aligned}$$

$$\bar{w} = 4\bar{u} - \frac{3}{2}\bar{v}$$

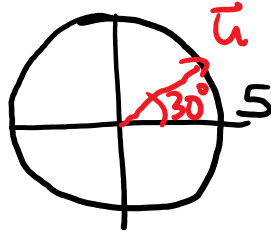
We'll do this algebraically in ch. 2

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

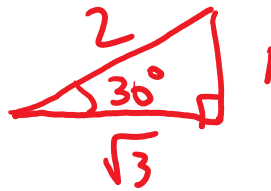
$$\vdots$$

Ex: TRIG

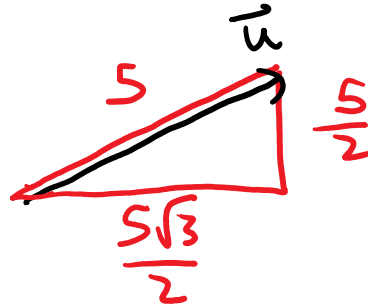
a)



Find \vec{u}

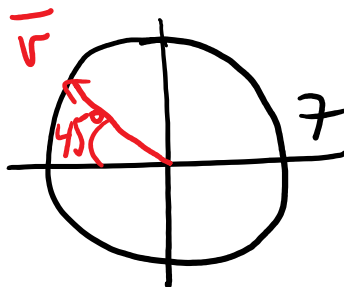


Multiply by $\frac{5}{2}$:



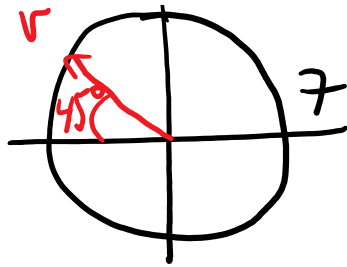
$$\vec{u} = \left[\frac{5\sqrt{3}}{2}, \frac{5}{2} \right]$$

b)

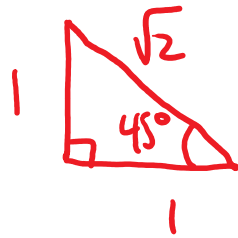


Find \vec{v}

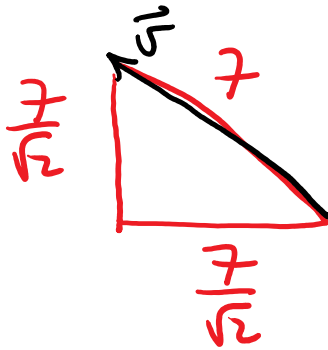
b)



Find \vec{r}



Multiply by $\frac{7}{\sqrt{2}}$:



$$\vec{r} = \left[-\frac{7}{\sqrt{2}}, \frac{7}{\sqrt{2}} \right] \quad \text{WATCH SIGNS}$$

$$\text{or } \left[-\frac{7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2} \right]$$