

www.leahhoward.com
Lecture Notes, Videos

Math Lab TEC 142

Coursepack : D2L

www.leahhoward.com/251CP.pdf

COURSE OVERVIEW : Geometry and Algebra

Geometry : lengths and angles
in 2D or 3D

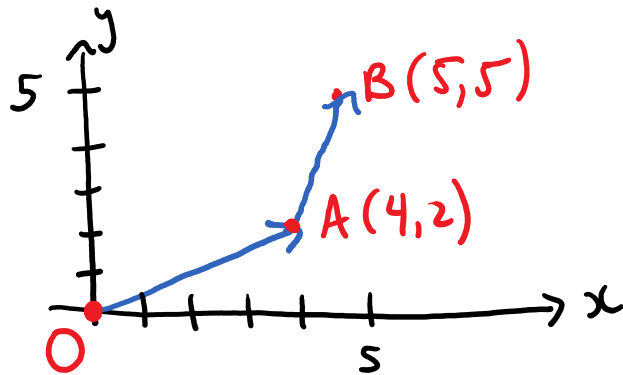
Algebra : formulas and calculations
in higher dimensions

e.g. tracking an object's position
and temperature is a 4D problem

1.1 Geometry and Algebra of Vectors

Vector: line segment with direction
Used for velocity, forces etc.

Ex:



Vectors $\vec{OA} = [4, 2]$

$\vec{AB} = [1, 3]$ Think $B-A$

↑ ↑
"Components"
of the vector

Sometimes we write

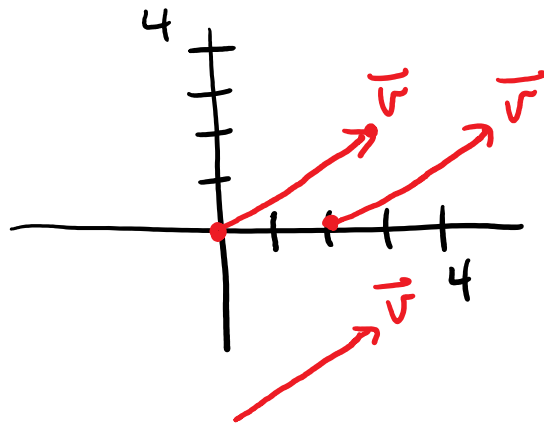
$$\vec{OA} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$$\vec{AB} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Ex: Points $C = (-1, -3)$ $D = (2, -1)$

Find $\vec{v} = \vec{CD}$ and draw it.

$$\vec{v} = [2 - (-1), -1 - (-3)] \text{ Think } D - C \\ = [3, 2]$$



A given vector can be drawn from any initial position.

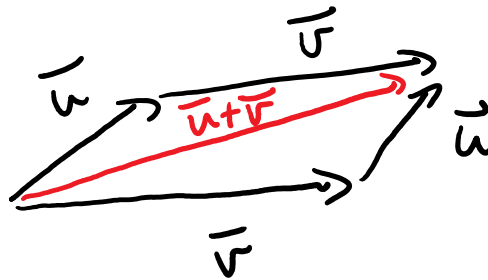
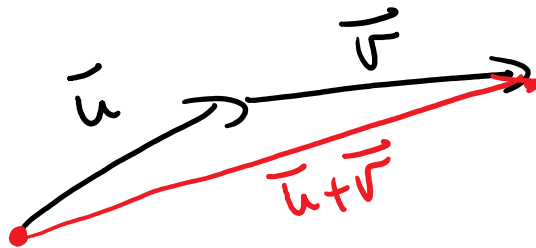
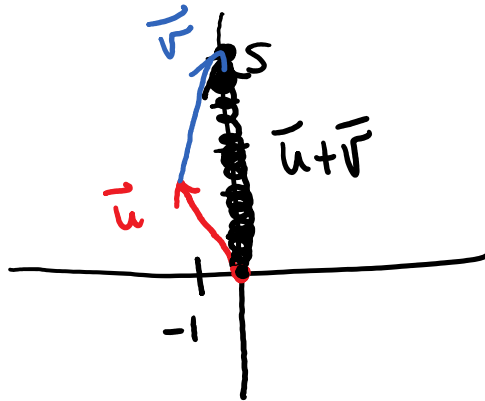
Rephrased: Same length and same direction
 \Rightarrow same vector

Notation: $[,]$ for vectors
 $(,)$ for points

Ex: Vector addition

$$\vec{u} = [-1, 2] \quad \vec{v} = [1, 3]$$

$$\begin{aligned}\vec{u} + \vec{v} &= [-1+1, 2+3] \\ &= [0, 5]\end{aligned}$$



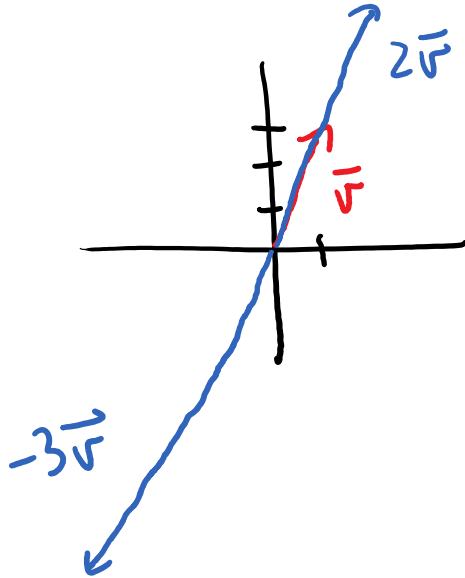
Ex: Scalar Multiplication

$$\vec{v} = [1, 3]$$

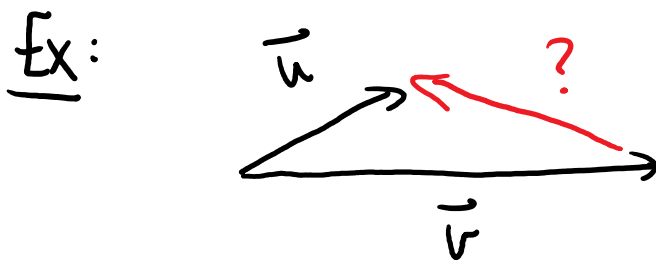
$$2\vec{v} = [2, 6]$$

$$-3\vec{v} = [-3, -9]$$

$\vec{v}, 2\vec{v}, -3\vec{v}$ are all parallel



Ex: $[2, 6] - [3, 4] = [2, 6] + [-3, -4]$
 $= [-1, 2]$

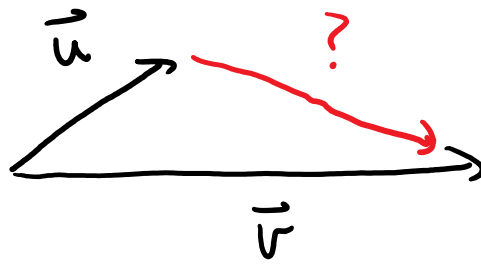


? = backwards along \vec{v} , then along \vec{u}

$$= -\vec{v} + \vec{u}$$

$$\propto \vec{u} - \vec{v}$$

Ex:



? = opposite of prev. example

$$= -(-\vec{v} + \vec{u})$$

$$= \vec{v} - \vec{u}$$

Alternatively: ? = backwards along \vec{u} ,
then along \vec{v}

$$= -\vec{u} + \vec{v}$$

$$\text{or } \vec{v} - \vec{u}$$

