Week 8 Tuesday

October 22, 2019 10:22 AM

Average > 80%

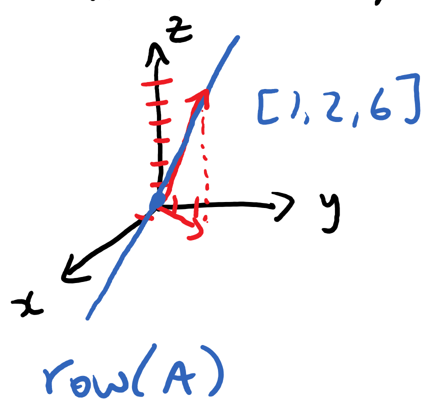
3.5 Subspaces and Dimension Cont'd

Ex: Visualization of row(A), col(A) and null(A)

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 4 & 12 \\ 0 & 0 & 0 \end{bmatrix}$$

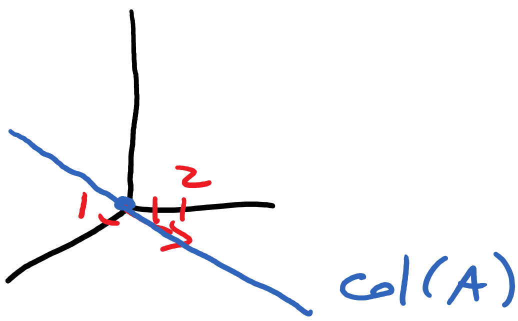
has RREF = $\begin{bmatrix} 1 & 2 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

a) Basis for row(A) = $\{ [1 \ 2 \ 6] \}$



b) Basis for col(A) = { col 1 of A }

$$= \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$$



c) Basis for null(A)

Solve $A\bar{x}=\bar{0}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{x_2 = s}$

$\boxed{x_3 = t}$

$x_1 + 2x_2 + 6x_3 = 0$

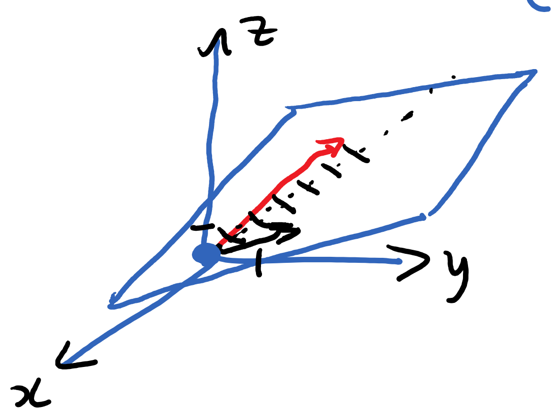
$\boxed{x_1 = -2s - 6t}$

$x_2 = 1s + 0t$

$$\vec{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} t$$

Basis for null(A) = $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 0 \\ 1 \end{bmatrix} \right\}$

null(A)



HANDOUT "Long Fundamental Theorem" (on website)

Statements a) to o) are all true or all false for any given square matrix.

a-e Section 3.3

f-g

h-j

k-m

n-o Chapter 4

Ex: Is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$ a basis for $\mathbb{R}^3$?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 1 & 1 & 4 \end{bmatrix}$$

$$\begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$R_3 + \tfrac{1}{3}R_2 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 3 \\ 0 & 0 & 2 \end{bmatrix} \quad \text{REF}$$

rank(A) = # nonzero rows in REF/RREF

$= 3$

f is true

$\Rightarrow$ all statements are true

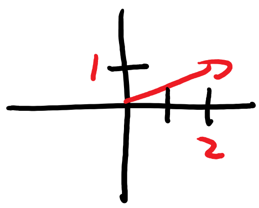
(in particular, m is true)

$\boxed{\text{Yes}}$

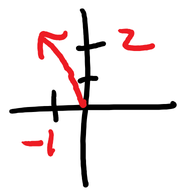
3.6 Linear Transformations

__Ex:__ Transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$

Rotates a vector $90°$ counterclockwise



apply T $\rightarrow$



__Notation:__
$$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

or

$$T\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Terminology: $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ is the **image** of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ under T

DEF

The **matrix transformation** T_A multiplies a vector on the left by A

i.e. $T_A(\vec{x}) = A\vec{x}$

Ex: a) $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}$

Find $T_A\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$

$$= A\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= \begin{bmatrix} 2x+z \\ -x+y+3z \end{bmatrix}$$

$T_A : \mathbb{R}^3 \to \mathbb{R}^2$

b) $T_A\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x+y \\ x-y \\ 3x+3y \end{bmatrix}$

Find A

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+y \\ x-y \\ 3x+3y \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 3 \end{bmatrix}$$

$$T_A : \mathbb{R}^2 \to \mathbb{R}^3$$