Week 3 Wednesday

September 18, 2019 8:45 AM

2.1 Cont'd

Terminology

$\begin{bmatrix} 2 & 6 \\ -3 & 3 \end{bmatrix}$ is a <u>coefficient</u> matrix

$\left[\begin{array}{cc|c} 2 & 6 & -14 \\ -3 & 3 & -15 \end{array}\right]$ is an <u>augmented</u> matrix

<u>Ex</u>: Solve $\begin{cases} 2x - 3y = 8 \\ -4x + 6y = -16 \end{cases}$

$$\begin{array}{ccc} x & y & \# \end{array}$$
$$\left[\begin{array}{cc|c} 2 & -3 & 8 \\ -4 & 6 & -16 \end{array}\right] \qquad \text{Goal: } \left[\begin{array}{cc|c} 1 & 0 & \\ 0 & 1 & \end{array}\right]$$

$R_1/2$ $\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 4 \\ -4 & 6 & -16 \end{array}\right]$

$$\begin{array}{ccc} x & y & \end{array}$$

$R_2 + 4R_1$ $\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 4 \\ 0 & 0 & 0 \end{array}\right] \leftarrow$ No INFO $0x + 0y = 0$

Current Row ± # (Pivot Row)

Column for y has no "1"

y is a "free variable"

$\boxed{y = t}$

$x - \frac{3}{2}y = 4 \rightarrow x = 4 + \frac{3}{2}y \rightarrow \boxed{x = 4 + \frac{3}{2}t}$

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$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} t$$

System has infinitely-many solutions

<u>Ex</u>: Solve

$$\begin{array}{cc} x & y & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 2 & 3 & 4 \\ 3 & 4 & 7 \end{array}\right]$$

$$\begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 3 & -6 \\ 0 & 4 & -8 \end{array}\right]$$

$$R_2/3 \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 4 & -8 \end{array}\right]$$

$$\begin{array}{cc} x & y \end{array}$$
$$R_3 - 4R_2 \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right] \leftarrow \begin{array}{c} 0 = 0 \\ \text{NO INFO} \end{array}$$

$1x + 0y = 5 \rightarrow x = 5$

$y = -2$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

<u>Ex</u>: Solve by "backsubstitution"

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 4 & 1 & 1 & 15 \\ 0 & 3 & 5 & 29 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

Start @ bottom: $2z = 8 \rightarrow \boxed{z=4}$

$$3y + 5z = 29 \rightarrow 3y + 20 = 29 \rightarrow \boxed{y=3}$$

$$4x + y + z = 15 \rightarrow 4x + 7 = 15 \rightarrow \boxed{x=2}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

2.2 Solving Systems

A matrix is in row-echelon form (REF) if:

1) any zero rows are at the bottom AND
2) the leading nonzero entries of each row move down and right

Ex: Matrices in REF

$$\left[\begin{array}{ccc|c} 6 & 0 & -1 & \\ 0 & 0 & 3 & \\ 0 & 0 & 0 & \end{array}\right] \qquad \left[\begin{array}{ccc|c} 2 & 3 & -1 & \\ 0 & 4 & 7 & \\ 0 & 0 & 0 & \end{array}\right]$$

Gaussian Elimination:

Transform coefficients to REF using the 3 row operations

Solve by back-substitution

Ex: Solve by Gaussian Elimination

$$\begin{array}{c} \begin{array}{ccc|c} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 2 & 2 & 0 & 8 \\ 0 & 3 & 1 & 8 \end{array}\right] \end{array}$$

Get 0's in Column 1

$$R_2 - 2R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & -2 & -2 & -4 \\ 0 & 3 & 1 & 8 \end{array}\right]$$

Get a 1:

$$R_2/(-2) \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 3 & 1 & 8 \end{array}\right]$$

$$R_3 - 3R_2 \quad \begin{array}{c} \begin{array}{ccc|c} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & -2 & 2 \end{array}\right] \end{array} \quad \text{REF}$$

Start @ bottom: $-2z = 2 \rightarrow \boxed{z = -1}$

$y + z = 2 \rightarrow y - 1 = 2 \rightarrow \boxed{y = 3}$

$$x + 2y + z = 6 \rightarrow x + 6 - 1 = 6 \rightarrow \boxed{x = 1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$