Week 3 Monday

September 16, 2019 7:57 AM

Cross Product Part 2

Matrix: rectangular array

e.g. $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 3 \end{bmatrix}$

Size: (#rows) × (#columns)

e.g. A is 2×3

The determinant of A is written $\det A$ or $|A|$. Only defined for square matrices.

FORMULAS

1) $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

2) $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} & \\ & \end{vmatrix} + c \begin{vmatrix} & \\ & \end{vmatrix}$

$\boxed{\text{Signs alternate according to } \begin{bmatrix} + & - & + \\ - & + & \\ + & & \ddots \end{bmatrix} \text{ "checkerboard pattern" for signs}}$

"checkerboard pattern" for signs

$$= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\begin{bmatrix} & \cancel{b} & \\ d & & f \\ g & & i \end{bmatrix}$$

Process is called "cofactor expansion"

Ex: Compute

a) $\det\begin{bmatrix} -1 & 4 & 6 \\ 2 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$

$$= -1\begin{vmatrix} 1 & 3 \\ 6 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 0 & 7 \end{vmatrix} + 6\begin{vmatrix} 2 & 1 \\ 0 & 6 \end{vmatrix}$$

$$= -1(-11) - 4(14) + 6(12)$$

$1(7)-3(6)$

$$= 27$$

b) $\begin{vmatrix} -1 & -4 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 8 \end{vmatrix}$

$$= -1\begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 1 & 8 \end{vmatrix} + 6\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= -1(6) + 4(6) + 6(0)$$
$$= 18$$

NOTATION

$$[1,0,0] = \vec{i}$$
$$[0,1,0] = \vec{j}$$
$$[0,0,1] = \vec{k}$$

2nd Method for Cross Product

$$[2,1,3] \times [-6,4,2] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 3 \\ -6 & 4 & 2 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & 3 \\ -6 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & 1 \\ -6 & 4 \end{vmatrix}$$

$$= \vec{i}(-10) - \vec{j}(22) + \vec{k}(14)$$

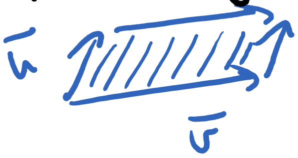
$$= -10[1,0,0] - 22[0,1,0] + 14[0,0,1]$$

$$= [-10, -22, 14]$$

3 Geometry Formulas

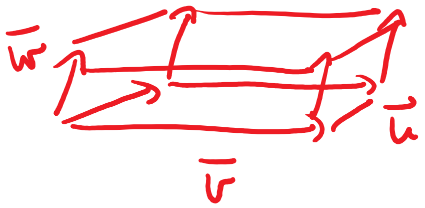
1) $A(\text{parallelogram in } \mathbb{R}^3) = \|\vec{u} \times \vec{v}\|$

2) A(parallelogram in $\mathbb{R}^2$) $= \left| \begin{vmatrix} \bar{u} \\ \bar{v} \end{vmatrix} \right|$



$\boxed{\text{absolute value of the determinant of } \bar{u}, \bar{v}}$

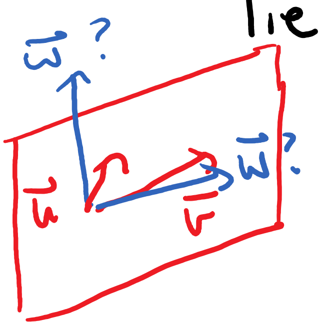
3) Slanted box or "parallelepiped"



V(parallelepiped in $\mathbb{R}^3$) $= \left| \begin{vmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{vmatrix} \right|$

$\boxed{\text{absolute value of the determinant of } \bar{u}, \bar{v}, \bar{w}}$

<u>Ex</u>: Do vectors [1,4,7], [2,5,9], [1,-2,-3] lie in a plane?



V(parallelepiped) = 0
if and only if $\bar{u}, \bar{v}, \bar{w}$ lie in a plane

$$V(\text{parallelepiped}) = \left| \begin{vmatrix} 1 & 4 & 7 \\ 2 & 5 & 9 \\ 1 & -2 & -3 \end{vmatrix} \right|$$

$$= \left| 1(3) - 4(-15) + 7(-9) \right|$$

$$= |0|$$

$$= 0$$

YES

$\underline{Ex}$: Area of parallelogram determined by [1, 6] and [3, 5]?

$$\text{Area} = \left| \begin{vmatrix} 1 & 6 \\ 3 & 5 \end{vmatrix} \right|$$

$$= |-13|$$

$$= 13$$