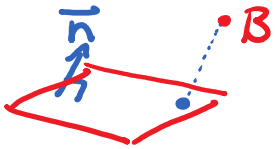
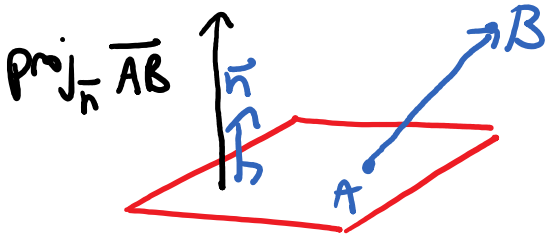
Week 2 Thursday

September 12, 2019 8:42 AM

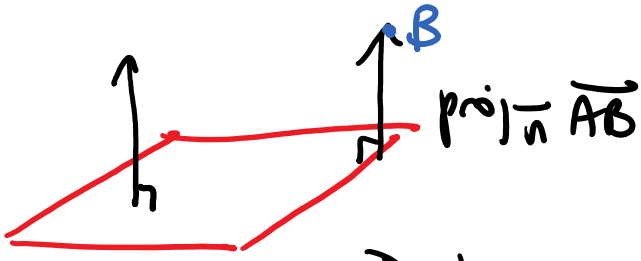
1.3 Cont'd

Ex: Find the distance between $B = (1, 3, 3)$ and the plane $P: x + y + 2z = 7$





Choose any point A on the plane



$$\text{Distance} = \| \text{proj}_{\vec{n}} \overrightarrow{AB} \|$$

Plane: $x + y + 2z = 7$

$A = (1, 2, 2)$ $B = (1, 3, 3)$

$$\overrightarrow{AB} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Normal vector $\vec{n} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

$$\text{proj}_{\vec{n}} \overrightarrow{AB} = \frac{\vec{n} \cdot \overrightarrow{AB}}{\|\vec{n}\|^2} \vec{n}$$

$$= \frac{3}{6} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{Distance} = \left\| \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\|$$

$$= \frac{1}{2} \left\| \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\|$$

$$= \frac{\sqrt{6}}{2}$$

Aside $\quad \vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$

Imagine that $\vec{u}, \vec{v}$ have length 1

$$\Rightarrow \vec{u} \cdot \vec{v} = \cos\theta$$

if $\vec{u}, \vec{v}$ have length 1

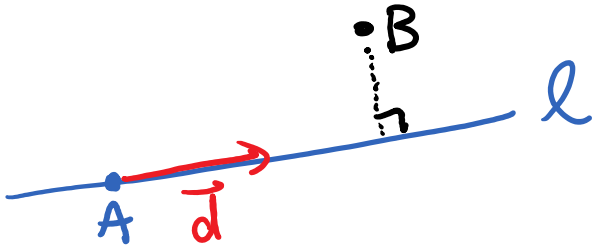
$\Rightarrow \quad \vec{u} \cdot \vec{v} = 1$

$\vec{u} \cdot \vec{v} = 0$

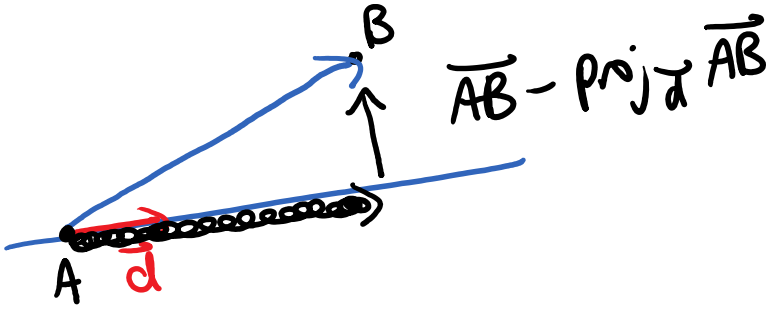
$\vec{u} \cdot \vec{v} = -1$

$\vec{u} \cdot \vec{v} = \cos\theta$

Ex: Find the distance between $B=(1,1,0)$ and the line ℓ through $A=(0,1,2)$ with $\vec{d}=\begin{bmatrix}1\\0\\1\end{bmatrix}$.



$\vec{d}$ = direction vector



$\Longrightarrow$ $= \text{proj}_{\vec{d}}\ \overrightarrow{AB}$

$$\text{Distance} = \| \overrightarrow{AB} - \text{proj}_{\vec{d}}\ \overrightarrow{AB} \|$$

$B=(1,1,0)$ $A=(0,1,2)$ $\vec{d}=\begin{bmatrix}1\\0\\1\end{bmatrix}$

$$\overrightarrow{AB}=\begin{bmatrix}1\\0\\-2\end{bmatrix}$$

$$\text{proj}_{\vec{d}}\ \overrightarrow{AB} = \frac{\vec{d}\cdot\overrightarrow{AB}}{\|\vec{d}\|^2}\,\vec{d}$$

$$= \frac{-1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$\overline{AB} - \text{proj}_{\vec{d}}\,\overline{AB} = \begin{bmatrix}1\\0\\-2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}3/2\\0\\-3/2\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}3\\0\\-3\end{bmatrix}$$

$$\text{Distance} = \| \overline{AB} - \text{proj}_{\vec{d}}\,\overline{AB} \|$$

$$= \| \frac{1}{2}\begin{bmatrix}3\\0\\-3\end{bmatrix} \|$$

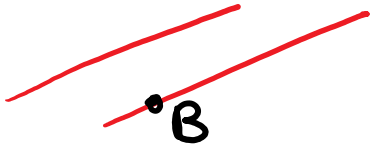
$$= \frac{1}{2} \| \begin{bmatrix}3\\0\\-3\end{bmatrix} \|$$

$$= \frac{\sqrt{18}}{2} \quad \text{or} \quad \frac{3\sqrt{2}}{2}$$

Comments:

1) For distance between parallel planes, choose any point on either plane as B

2) For distance between parallel lines, choose any point on either line as B.



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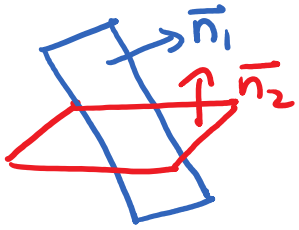
The angle between planes is defined as the angle between their normals.



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Parallel planes have parallel normals
Orthogonal " orthogonal