# Week 2 Monday

September 9, 2019 7:53 AM

## 1.2 Length and Angle Cont'd

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$ for $\vec{u}, \vec{v}$ in $\mathbb{R}^n$

Ex: Find the angle between
$\vec{u} = [1, -1, 1, 4]$ and $\vec{v} = [2, 3, 2, 1]$

$$\vec{u} \cdot \vec{v} = 1(2) + (-1)(3) + 1(2) + 4(1)$$
$$= 5$$

$$\|\vec{u}\| = \sqrt{1+1+1+16} = \sqrt{19}$$

$$\|\vec{v}\| = \sqrt{4+9+4+1} = \sqrt{18}$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$5 = \sqrt{19}\sqrt{18} \cos\theta$$

$$\frac{5}{\sqrt{19}\sqrt{18}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{19}\sqrt{18}}\right)$$

$$\approx 74^\circ$$

$\bar{u} \cdot \bar{v} > 0$

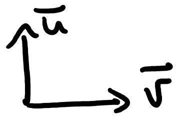
$\cos\theta > 0$

$0^\circ \le \theta < 90^\circ$


$\bar{u} \cdot \bar{v} = 0$

$\cos\theta = 0$

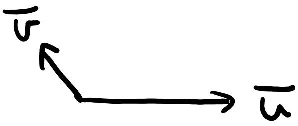
$\theta = 90^\circ$




$\bar{u} \cdot \bar{v} < 0$

$\cos\theta < 0$

$90^\circ < \theta \le 180^\circ$



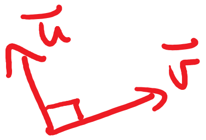

**Def**

$\bar{u}$ and $\bar{v}$ are <u>orthogonal</u> if $\bar{u} \cdot \bar{v} = 0$

Equivalent statements:

$\bar{u}$ and $\bar{v}$ are <u>perpendicular</u> (geometry language)

$\bar{u}$ and $\bar{v}$ are <u>orthogonal</u> (algebra language)

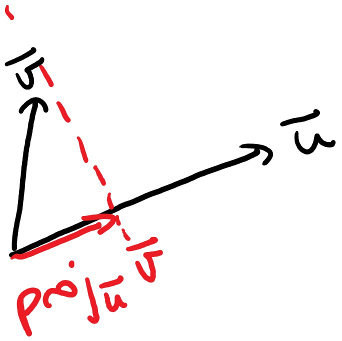



The projection of $\bar{v}$ onto $\bar{u}$ is written $\text{proj}_{\bar{u}}\,\bar{v}$

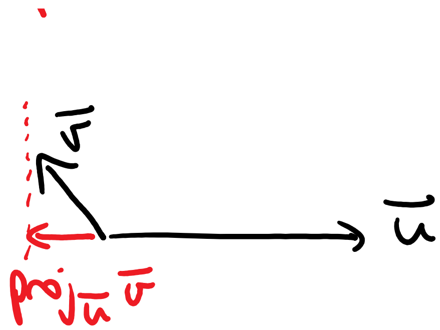
Could be read as "projection onto $\bar{u}$ of $\bar{v}$"

Quick Ex:

a)

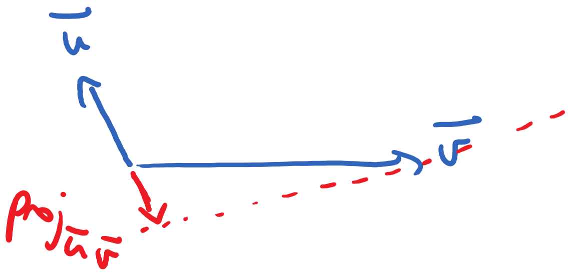


b)




c)


d)

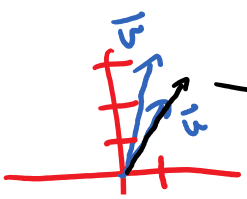



$$\boxed{\begin{array}{l}\text{Projection onto } \vec{u} \text{ of } \vec{v} \\ \qquad \text{proj}_{\vec{u}}\,\vec{v} = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\,\vec{u} \quad \text{in } \mathbb{R}^n\end{array}}$$

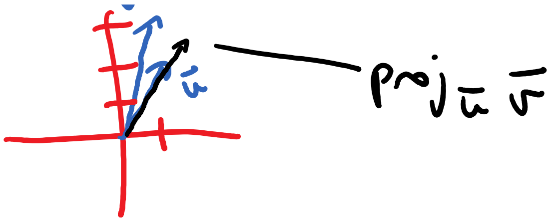
<u>Ex</u>: Find $\text{proj}_{\vec{u}}\,\vec{v}$ for $\vec{u} = [1,2]$, $\vec{v} = [1,3]$

$$\text{proj}_{\vec{u}}\,\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\,\vec{u}$$

$$= \frac{7}{5}[1,2] \quad \text{or} \quad \frac{7}{5}\vec{u}$$

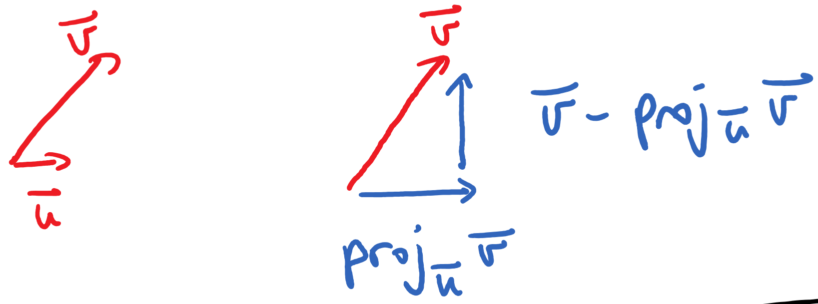



$\text{proj}_{\vec{u}}\,\vec{v}$



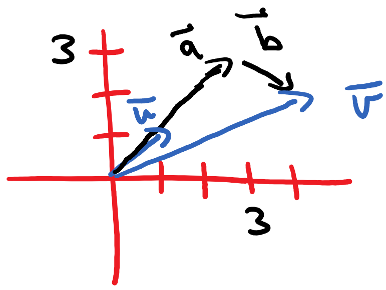

FACT

Given vectors $\vec{u}$ and $\vec{v}$, we can decompose $\vec{v}$ into vectors parallel and perpendicular to $\vec{u}$.




Ex: $\vec{u} = [1, 1]$ $\vec{v} = [4, 2]$

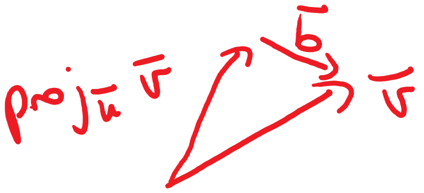
Find $\vec{a}$ and $\vec{b}$ so that $\vec{v} = \vec{a} + \vec{b}$, $\vec{a}$ is parallel to $\vec{u}$ and $\vec{b}$ is perpendicular to $\vec{u}$.




$$\vec{a} = \text{proj}_{\vec{u}}\vec{v}$$
$$= \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\vec{u}$$
$$= \frac{6}{2}[1, 1]$$

$= 3[1,1]$

$= [3,3]$



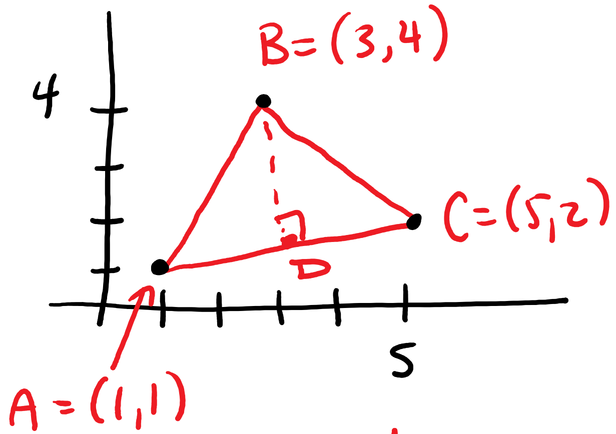

$$\text{proj}_{\bar{u}}\bar{v} + \bar{b} = \bar{v}$$

$$\bar{b} = \bar{v} - \text{proj}_{\bar{u}}\bar{v}$$

$$= [4,2] - [3,3]$$

$$= [1,-1]$$

Ex:




Find D