Week 14 Wednesday

December 4, 2019 8:41 AM

(10) $\text{span}(\bar{u}+\bar{v}, \bar{u}-\bar{v}, \bar{u}+\bar{v}+\bar{w})$

$$= \{ c_1(\bar{u}+\bar{v}) + c_2(\bar{u}-\bar{v}) + c_3(\bar{u}+\bar{v}+\bar{w}) \}$$

Notice $1(\bar{u}+\bar{v}) + 1(\bar{u}-\bar{v}) = 2\bar{u}$

Conclude $\bar{u} = \frac{1}{2}(\bar{u}+\bar{v}) + \frac{1}{2}(\bar{u}-\bar{v})$ ✓

Notice $1(\bar{u}+\bar{v}) - 1(\bar{u}-\bar{v}) = 2\bar{v}$

Conclude $\frac{1}{2}(\bar{u}+\bar{v}) - \frac{1}{2}(\bar{u}-\bar{v}) = \bar{v}$ ✓

$$\bar{w} = \boxed{-1}(\bar{u}+\bar{v}) + \boxed{0}(\bar{u}-\bar{v}) + \boxed{1}(\bar{u}+\bar{v}+\bar{w})$$ ✓

(12) Let $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be in the span.

$$c_1\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + c_2\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} + c_3\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 1 & 1 & 0 & b \\ 1 & 0 & 0 & c \\ 1 & 0 & 1 & d \end{array}\right]$$

Each zero row in the REF will produce a condition on a, b, c, d

$$\sim \left[\begin{array}{ccc|c} & & & \\ & & & \\ 0 & 0 & 0 & b-a \end{array}\right]$$

3 nonzero rows

Conclude $b-a=0$

$b=a$

$$\text{span} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \,\middle|\, b=a \right\}$$

$$= \left\{ \begin{bmatrix} a & a \\ c & d \end{bmatrix} \right\}$$

(14)

$$E_3 E_2 E_1 A = I$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underbrace{E_3 E_2 E_1}_{A^{-1}} A = I$$

$$A^{-1} = E_3 E_2 E_1$$

$$A = (A^{-1})^{-1}$$
$$= E_1^{-1} E_2^{-1} E_3^{-1}$$

Socks and shoes

$$= \underset{R_2 \leftrightarrow R_3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}} \underset{2R_3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}} \underset{R_1 \to R_1 + 3R_3}{\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

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a) row(A)

= {nonzero rows of REF/RREF}

$$= \{ [1 \ 0 \ -1 \ -2], [0 \ 1 \ 2 \ 3] \}$$

b) row(A) = col(A^T)

Use columns 1 and 3 of A^T

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

or $\{ [1 \ 2 \ 3 \ 4], [1 \ 1 \ 1 \ 1] \}$

c) col (A)

Use columns 1 and 2 of A

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 1 \\ 4 \end{bmatrix} \right\}$$

d) null(A)

Solve $A\vec{x} = \vec{0}$

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ & \cdot & \cdot & \cdot & 0 \\ & & & & 0 \\ & & & & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

RREF

$x_3 = s$ $x_4 = t$

$x_1 - x_3 - 2x_4 = 0 \rightarrow x_1 = s + 2t$

$x_2 + 2x_3 + 3x_4 = 0 \rightarrow x_2 = -2s - 3t$

$$\vec{x} = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} t$$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

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Section Reference for Review Problems

#	Section	#	Section	#	Section
1	1.2	13	3.3	25	4.4
2	1.2	14	3.3	26	5.1
3	1.3	15	3.4	27	5.2
4	1.3	16	3.5	28	5.2
5	1.3	17	3.5	29	5.2
6	Cross Product	18	3.6	30	5.3
7	2.2	19	3.6	31	5.4
8	2.2	20	3.6	32	7.3
9	2.2	21	4.2	33	7.3
10	2.3	22	4.2 / 3.5	34	Complex
11	2.3	23	4.3	35	Complex
12	3.2	24	4.4	36	Complex