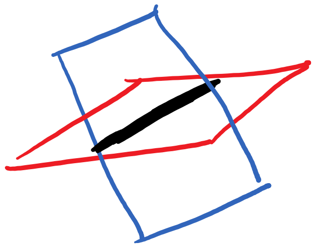
Week 14 Monday

December 2, 2019 7:49 AM

Exam Review

Ex: Find parametric equations for the intersection of $2x+4y+8z=10$ and $2x+5y+5z=7$



$$\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 2 & 4 & 8 & 10 \\ 2 & 5 & 5 & 7 \end{array}\right] \end{array}$$

$R_1/2$ $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 2 & 5 & 5 & 7 \end{array}\right]$

R_2-2R_1 $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & -3 \end{array}\right]$

R_1-2R_2 $\begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 10 & 11 \\ 0 & 1 & -3 & -3 \end{array}\right] \end{array}$ RREF

$z=t$

$x+10z=11 \rightarrow x = 11-10t$

$y-3z=-3 \rightarrow y = -3+3t$

Parametric Form

$$\begin{cases} x = 11 - 10t \\ y = -3 + 3t \\ z = t \end{cases}$$

Vector Form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -10 \\ 3 \\ 1 \end{bmatrix} t$$

<u>Ex:</u> Find A^{-1} for $A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & -1 & 1 \\ -3 & 3 & 4 \end{bmatrix}$

$[A \mid I]$

$\rightsquigarrow [I \mid A^{-1}]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -3 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 + 3R_1 \quad \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array}\right]$$

$$R_2/(-1) \quad \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array}\right]$$

R_1+R_2 $\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array}\right]$

R_1+2R_3
R_2+R_3 $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7 & -1 & 2 \\ 0 & 1 & 0 & 3 & -1 & 1 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array}\right]$ ← A^{-1}

Ex: Show that $\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ is an orthogonal basis for $\mathbb{R}^3$.

Show that:

a) the set is orthogonal

b) the set is a basis for $\mathbb{R}^3$

a) Call vectors $\vec{u}, \vec{v}, \vec{w}$

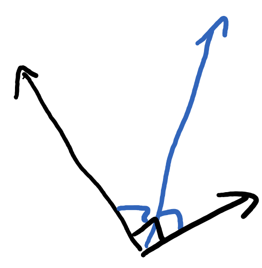
$\vec{u} \cdot \vec{v} = 0$ ✓

$\vec{u} \cdot \vec{w} = 0$ ✓

$\vec{v} \cdot \vec{w} = 0$ ✓

b) Section 5.1

n nonzero orthogonal vectors form a basis for $\mathbb{R}^n$



Alternatively : $\begin{vmatrix} 0 & 1 & 2 \\ 0 & -2 & 1 \\ 1 & 0 & 0 \end{vmatrix} \neq 0$

$\Rightarrow$ vectors form a basis for $\mathbb{R}^3$

<u>Ex</u>: Find $\text{proj}_W \vec{u}$ where

$W = \text{span}\left(\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} \right)$ and $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix}$

Basis is not <u>orthogonal</u>

Need Gram-Schmidt

Partial basis $X = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

$$\vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} - \text{proj}_{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} - \frac{4}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$3\vec{v}_2 = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 0 \end{bmatrix} - 4 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 5 \\ -4 \end{bmatrix}$$

<u>Orthogonal</u> Basis for $W = \left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}}_{\vec{w}_1}, \underbrace{\begin{bmatrix} -1 \\ 6 \\ 5 \\ -4 \end{bmatrix}}_{\vec{w}_2} \right\}$

$$\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{aligned} \text{proj}_W \vec{u} &= \text{proj}_{\vec{w}_1} \vec{u} + \text{proj}_{\vec{w}_2} \vec{u} \\ &= \frac{6}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + \frac{27}{78} \begin{bmatrix} -1 \\ 6 \\ 5 \\ -4 \end{bmatrix} \end{aligned}$$

Ex: Write $(-1+4i)^8$ in the form $a+bi$.

To Be Continued...