Week 1 Thursday

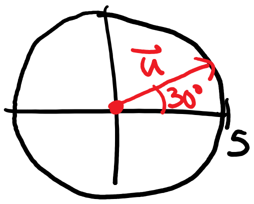
September 5, 2019 8:38 AM

Coursepack on D2L

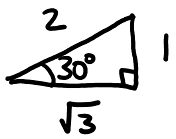
Which problems to do? Course Outline or Course Website

1.1 Cont'd

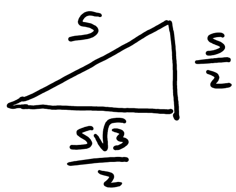
Ex: a)



Find $\vec{u}$

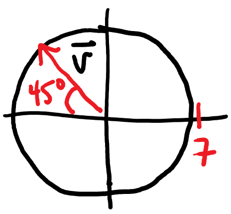


Multiply by $\frac{5}{2}$:



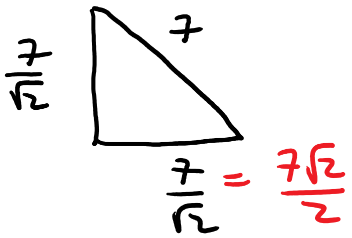
$$\vec{u} = \left[\frac{5\sqrt{3}}{2}, \frac{5}{2}\right]$$

b)



Find $\vec{v}$

Multiply by $\frac{7}{\sqrt{2}}$:



$\bar{v} = \left[-\frac{7\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right]$

1.2 Length and Angle

$\bar{u} = [1, 4, 2, -9] \quad \bar{v} = [2, 3, -2, -1]$

Dot Product $\quad \bar{u} \cdot \bar{v} = 1(2) + 4(3) + 2(-2) + (-9)(-1)$

$= 19$

Ex: a) $[1, 5] \cdot [2, -3] = 1(2) + 5(-3)$

$= -13$

b) $[1, 5] \cdot [2, 3, 0]$ is undefined

c) $[u_1, u_2] \cdot [u_1, u_2] = u_1^2 + u_2^2$

FACT 3 Properties of Dot Product:

1) $\vec{u}\cdot\vec{u} \geq 0$

2) $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$ for all $\vec{u},\vec{v}$

3) $\vec{u}\cdot\vec{u} = 0$ if and only if $\vec{u} = \vec{0}$

- if $\vec{u}\cdot\vec{u} = 0$ then $\vec{u} = \vec{0}$ (less obvious)
- if $\vec{u} = \vec{0}$ then $\vec{u}\cdot\vec{u} = 0$ (obvious)

Ex: Simplify

a) $(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v})$

$= \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}$

$= \vec{u}\cdot\vec{u} + 2\,\vec{u}\cdot\vec{v} + \vec{v}\cdot\vec{v}$

b) $3\vec{u}\cdot(-2\vec{v}+5\vec{w})$

$= -6\,\vec{u}\cdot\vec{v} + 15\,\vec{u}\cdot\vec{w}$

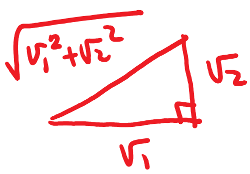
Caution: $\vec{u}\vec{v}$ is nonsense

DEF

The length or norm of $\vec{v} = [v_1, v_2, \ldots, v_n]$ is

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

Comes from



Ex: a) $\|[1,1,1,-2]\| = \sqrt{1+1+1+4} = \sqrt{7}$

b) $\|[3,-1]\| = \sqrt{9+1} = \sqrt{10}$

Notice $[3,-1]\cdot[3,-1] = 3(3) + (-1)(-1)$
$= 10$

FACT

$$\vec{v}\cdot\vec{v} = \|\vec{v}\|^2 \quad \text{for any } \vec{v}$$

Ex: Let $\vec{v} = [v_1, v_2, v_3]$

Simplify $\|-3\vec{v}\|$

$$= \|[-3v_1, -3v_2, -3v_3]\|$$

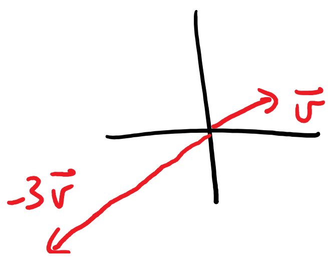
$$= \sqrt{(-3v_1)^2 + (-3v_2)^2 + (-3v_3)^2}$$

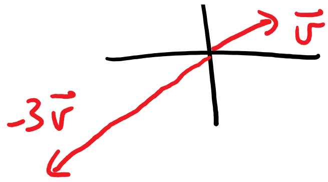
$$= \sqrt{9v_1^2 + 9v_2^2 + 9v_3^2}$$

$$= \sqrt{9(v_1^2 + v_2^2 + v_3^2)}$$

$$= 3\sqrt{v_1^2 + v_2^2 + v_3^2}$$

$$= 3\|\vec{v}\|$$





$\|-3\vec{v}\| = 3\|\vec{v}\|$

"The length of $-3\vec{v}$ is 3 times the length of $\vec{v}$"

FACT

$$\|c\vec{v}\| = |c|\,\|\vec{v}\| \text{ for any } \vec{v}$$