① $|A_1| = \begin{vmatrix} -115 & 27 \\ -958 & 45 \end{vmatrix} = 20691$

$|A_2| = \begin{vmatrix} 38 & -115 \\ 23 & -958 \end{vmatrix} = -33759$

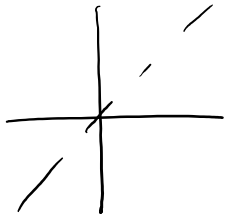
$|A| = \begin{vmatrix} 38 & 27 \\ 23 & 45 \end{vmatrix} = 1089$

$x = \frac{|A_1|}{|A|} = 19 \qquad y = \frac{|A_2|}{|A|} = -31$

② $[T_1] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}_{\theta=-150°}$

$= \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

$[T_2] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



$[T] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

$\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$

$$= \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

③ a) $\left\{ \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 4 \end{bmatrix} \right\}$

b) Solve $A\bar{x} = \bar{0}$

$$\left[\begin{array}{ccc|c} 1 & 0 & -6 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3 = t$

$x_1 - 6x_3 = 0 \rightarrow x_1 = 6t$

$x_2 + 5x_3 = 0 \rightarrow x_2 = -5t$

$$\bar{x} = \begin{bmatrix} 6 \\ -5 \\ 1 \end{bmatrix} t$$

Basis for null(A) = $\left\{ \begin{bmatrix} 6 \\ -5 \\ 1 \end{bmatrix} \right\}$

c) $[72, k, 12] = [6, -5, 1]\,t$

1st component $\Rightarrow t = 12$

$\Rightarrow k = -60$

④ Orthogonal basis $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$

$$\Rightarrow \quad \bar{v} = \text{proj}_{\bar{b}_1} \bar{v} + \text{proj}_{\bar{b}_2} \bar{v} + \text{proj}_{\bar{b}_3} \bar{v}$$

$$= \frac{\bar{v} \cdot \bar{b}_1}{\|\bar{b}_1\|^2} \bar{b}_1 + \dots$$

$$= \frac{20}{14} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{6}{6} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \frac{18}{21} \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix}$$

(5) $$P^{-1} = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$P^{-1} A P = D$$

$$A = P D P^{-1}$$

$$A^n = P D^n P^{-1}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} (-3)^n & 0 \\ 0 & 3^n \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \cdot (-3)^n & -(-3)^n \\ 3^n & 3^n \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} & (-3)^n + 2 \cdot 3^n \end{bmatrix}$$

$$\text{Answer} = \frac{2 \cdot 3^n + (-3)^n}{3}$$