1. $\det(7A^T B^{-1})$

$$= 7^3 \det(A^T B^{-1})$$
$$= 7^3 \det(A^T) \det(B^{-1})$$
$$= 7^3 \det A \cdot \frac{1}{\det B}$$
$$= 943.25$$

2. a) Find a basis for Col(A^T)

$$= \left\{ \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} \right\}$$

b) Solve $A^T \bar{x} = \bar{0}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_2 = t$

$x_1 + 2x_2 = 0 \rightarrow x_1 = -2t$

$x_3 = 0$

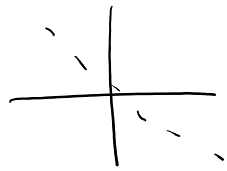
$$\bar{x} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} t$$

Basis for null$(A^T) = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

c) A line since it has 1 basis vector.

(3)

$$[T_1] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$



$$[T_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \theta = -120^\circ$$

$$= \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

$$[T] = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

(4)

$$P^{-1} = \frac{1}{8} \begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$P^{-1}AP = D$$

$$A = PDP^{-1}$$

$$A^n = PD^nP^{-1}$$

$$= \frac{1}{8} \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 5^n & 0 \\ 0 & (-3)^n \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -3 & 5 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 5^n & 5^n \\ (-3)^{n+1} & 5(-3)^n \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} & 3\cdot 5^n + 5(-3)^n \end{bmatrix}$$

$$\text{Answer} = \frac{3\cdot 5^n + 5(-3)^n}{8}$$

(5) Orthogonal basis $\{\bar{b}_1, \bar{b}_2, \bar{b}_3\}$

$$\Rightarrow \quad \bar{v} = \text{proj}_{\bar{b}_1}\bar{v} + \text{proj}_{\bar{b}_2}\bar{v} + \text{proj}_{\bar{b}_3}\bar{v}$$

$$= \frac{\bar{v}\cdot\bar{b}_1}{\|\bar{b}_1\|^2}\,\bar{b}_1 + \cdots$$

$$= \frac{6}{14}\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + \frac{11}{3}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - \frac{16}{42}\begin{bmatrix} -5 \\ -1 \\ 4 \end{bmatrix}$$