

Math 251-DX02
Test 3

SUBMISSION DEADLINE: 4:30pm Pacific Time
Submit on D2L

Number of Questions: 5
Total Marks: 18

Show all your work for full marks.

You MAY use the course website (notes, videos etc) and your own notes

You may NOT copy from others (classmates, tutors, Google, Chegg etc)

Submit jpg or pdf files

Feel free to handwrite your solutions and take photos of your work

1. [4 marks] Solve the system using Cramer's Rule:

$$38x + 27y = -115$$

$$23x + 45y = -958$$

2. [3 marks] Let T be the transformation from \mathbb{R}^2 to \mathbb{R}^2 that first rotates a vector by 150° clockwise then reflects it in the line $y = x$. Find the standard matrix for T .

3. [3 marks] Consider $A = \begin{bmatrix} 3 & 4 & 2 \\ 6 & 8 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ and $A^T = \begin{bmatrix} 3 & 6 & 2 \\ 4 & 8 & 4 \\ 2 & 4 & 8 \end{bmatrix}$.

The RREF of A is $\begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$. The RREF of A^T is $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

- a) Find a basis for the column space of A .
b) Find a basis for the null space of A .
c) Find k so that $[72, k, 12]$ is in the null space of A .

4. [3 marks] Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} \right\}$ be an orthogonal basis for \mathbb{R}^3 . Write $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix}$ as a linear combination of the basis vectors.

5. [5 marks] $P = \begin{bmatrix} 1 & 1 \\ -1 & 2 \end{bmatrix}$ diagonalizes A to produce $D = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$. Find the bottom-right entry of A^n (where n is a positive integer).