

Math 251-DX01

Test 3

SUBMISSION DEADLINE: 3:30pm Pacific Time

Submit on D2L

Number of Questions: 5

Total Marks: 18

Show all your work for full marks.

You MAY use the course website (notes, videos etc) and your own notes

You may NOT copy from others (classmates, tutors, Google, Chegg etc)

Submit jpg or pdf files

Feel free to handwrite your solutions and take photos of your work

1. [4 marks]  $A$  and  $B$  are  $3 \times 3$  matrices such that  $\det A = 11$  and  $\det B = 4$ . Find  $\det(7A^T B^{-1})$ .

2. [3 marks] Consider  $A = \begin{bmatrix} 3 & 4 & 2 \\ 6 & 8 & 4 \\ 2 & 4 & 8 \end{bmatrix}$  and  $A^T = \begin{bmatrix} 3 & 6 & 2 \\ 4 & 8 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ .

The RREF of  $A$  is  $\begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$ . The RREF of  $A^T$  is  $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ .

- Find a basis for the row space of  $A$ , consisting of rows of  $A$ .
- Find a basis for the null space of  $A^T$ .
- Is the null space of  $A^T$  a point, a line or a plane? Explain briefly.

3. [3 marks] Let  $T$  be the transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  that first reflects a vector in the line  $y = -x$  then rotates it by  $120^\circ$  clockwise. Find the standard matrix for  $T$ .

4. [5 marks]  $P = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$  diagonalizes  $A$  to produce  $D = \begin{bmatrix} 5 & 0 \\ 0 & -3 \end{bmatrix}$ . Find the bottom-right entry of  $A^n$  (where  $n$  is a positive integer).

5. [3 marks] Let  $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 4 \end{bmatrix} \right\}$  be an orthogonal basis for  $\mathbb{R}^3$ . Write  $\mathbf{v} = \begin{bmatrix} 6 \\ -2 \\ 3 \end{bmatrix}$  as a linear combination of the basis vectors.