1. $x(C_4H_6O_3) + y(H_2O) \to z(C_2H_4O_2)$

$$\left.\begin{aligned} C &: \quad 4x = 2z \\ H &: \quad 6x + 2y = 4z \\ O &: \quad 3x + y = 2z \end{aligned}\right\}$$

2. Let $c_1\bar{u} + c_2\bar{v} = \bar{w}$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & 44 \\ -2 & 1 & -13 \\ 3 & 4 & 107 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 + 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 44 \\ 0 & 5 & 75 \\ 0 & -2 & -25 \end{array}\right]$$

$$\frac{R_2}{5} \left[\begin{array}{cc|c} 1 & 2 & 44 \\ 0 & 1 & 15 \\ 0 & -2 & -25 \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 + 2R_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 14 \\ 0 & 1 & 15 \\ 0 & 0 & 5 \end{array}\right]$$

No solution

Not possible to write $\bar{w}$ as $c_1\bar{u} + c_2\bar{v}$

③ $L \underbrace{U\bar{x}}_{\bar{y}} = \bar{b}$

1) Solve $L\bar{y} = \bar{b}$

$$\begin{array}{ccc} y_1 & y_2 & y_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ -2 & 1 & 0 & 6 \\ 3 & 4 & 1 & -25 \end{array}\right]$$

$$y_1 = -2$$

$$-2y_1 + y_2 = 6 \rightarrow 4 + y_2 = 6 \rightarrow y_2 = 2$$

$$3y_1 + 4y_2 + y_3 = -25 \rightarrow -6 + 8 + y_3 = -25 \rightarrow y_3 = -27$$

2) Solve $U\bar{x} = \bar{y}$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 3 & 1 & -3 & -2 \\ 0 & 2 & 6 & 2 \\ 0 & 0 & -9 & -27 \end{array}\right]$$

$$-9x_3 = -27 \rightarrow x_3 = 3$$

$$2x_2 + 6x_3 = 2 \rightarrow 2x_2 + 18 = 2 \rightarrow x_2 = -8$$

$$3x_1 + x_2 - 3x_3 = -2 \rightarrow 3x_1 - 8 - 9 = -2 \rightarrow x_1 = 5$$

$$\bar{x} = \begin{bmatrix} 5 \\ -8 \\ 3 \end{bmatrix}$$

④ $E_1: \quad R_2 \to R_2 - 3R_1 \qquad E_1 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$E_2: \quad \frac{R_1}{4} \qquad E_2 = \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & 1 \end{bmatrix}$

$$\underbrace{E_2 E_1}_{A^{-1}} A = I$$

$$\begin{aligned} A &= (A^{-1})^{-1} \\ &= E_1^{-1} E_2^{-1} \\ &= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

⑤ $A^{-1} = \frac{1}{11} \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$

$$22A^{-1} - A^T + BC$$

$$= 2\begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 33 & -2 \\ 4 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} 39 & 3 \\ 5 & 36 \end{bmatrix}$$