

$$x^2 + y^2 + ax + by + c = 0$$

①

$$\left. \begin{array}{l} \text{Sub } (-1, -3) : \quad 10 - a - 3b + c = 0 \\ \text{Sub } (4, 2) : \quad 20 + 4a + 2b + c = 0 \\ \text{Sub } (9, -3) : \quad 90 + 9a - 3b + c = 0 \end{array} \right\}$$

②

$$\text{Let } C_1 \bar{u} + C_2 \bar{v} = \bar{w}$$

$$\begin{array}{c} C_1 \quad C_2 \\ \left[\begin{array}{cc|c} 1 & 2 & 47 \\ -2 & 1 & -9 \\ 3 & 4 & 107 \end{array} \right] \end{array}$$

$$\begin{array}{l} R_2 + 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 47 \\ 0 & 5 & 85 \\ 0 & -2 & -34 \end{array} \right]$$

$$\frac{R_2}{5} \left[\begin{array}{cc|c} 1 & 2 & 47 \\ 0 & 1 & 17 \\ 0 & -2 & -34 \end{array} \right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 + 2R_2 \end{array} \begin{array}{c} C_1 \quad C_2 \\ \left[\begin{array}{cc|c} 1 & 0 & 13 \\ 0 & 1 & 17 \\ 0 & 0 & 0 \end{array} \right] \end{array}$$

$$C_1 = 13 \quad C_2 = 17$$

$$\bar{w} = 13 \bar{u} + 17 \bar{v}$$

③

$$L \underbrace{U}_{\bar{y}} \bar{x} = \bar{b}$$

1) Solve $L\bar{y} = \bar{b}$

$$\begin{array}{c} y_1 \quad y_2 \quad y_3 \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ -4 & 1 & 0 & 7 \\ 2 & 3 & 1 & -65 \end{array} \right] \end{array}$$

$$y_1 = -5$$

$$-4y_1 + y_2 = 7 \rightarrow 20 + y_2 = 7 \rightarrow y_2 = -13$$

$$2y_1 + 3y_2 + y_3 = -65 \rightarrow -10 - 39 + y_3 = -65 \rightarrow y_3 = -16$$

2) Solve $U\bar{x} = \bar{y}$

$$\begin{array}{c} x_1 \quad x_2 \quad x_3 \\ \left[\begin{array}{ccc|c} 2 & 1 & -2 & -5 \\ 0 & 3 & 7 & -13 \\ 0 & 0 & -8 & -16 \end{array} \right] \end{array}$$

$$-8x_3 = -16 \rightarrow x_3 = 2$$

$$3x_2 + 7x_3 = -13 \rightarrow 3x_2 + 14 = -13 \rightarrow x_2 = -9$$

$$2x_1 + x_2 - 2x_3 = -5 \rightarrow 2x_1 - 9 - 4 = -5 \rightarrow x_1 = 4$$

$$\bar{x} = \begin{bmatrix} 4 \\ -9 \\ 2 \end{bmatrix}$$

④

$$E_1: \frac{R_2}{3}$$

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$E_2: R_1 \rightarrow R_1 + 4R_2$$

$$E_2 = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$$

$$\underbrace{E_2 E_1}_A A = I$$

$$\begin{aligned} A &= (A^{-1})^{-1} \\ &= E_1^{-1} E_2^{-1} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$\textcircled{5} \quad A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$$

$$15A^{-1} - A^T + BC$$

$$= 3 \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 30 & -2 \\ 0 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 38 & 3 \\ -1 & 33 \end{bmatrix}$$