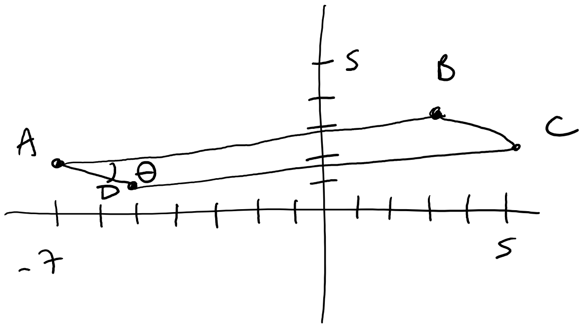
1



$$\bar{u} = \overrightarrow{AB} = \begin{bmatrix} 10 \\ 1 \end{bmatrix}$$

$$\bar{v} = \overrightarrow{AD} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \, \|\bar{v}\|} \right)$$

$$= \cos^{-1}\left(\frac{19}{\sqrt{101}\sqrt{5}} \right)$$

$$\approx 32^\circ$$

2 $\quad V(\text{parallelepiped}) = \left| \begin{vmatrix} 2 & -6 & 3 \\ 4 & 5 & -1 \\ c & 3 & 4 \end{vmatrix} \right|$

$$= \left| \, 2(23) + 6(16 + c) + 3(12 - 5c) \, \right|$$

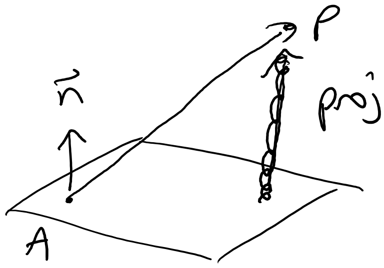
$$= |178 - 9c|$$

Vectors are coplanar if and only if $V(\text{parallelepiped}) = 0$

$$178 - 9c = 0$$

$$c = \frac{178}{9}$$

(3)



A = any point on plane

$A = (5, 0, 0)$

$$\overline{AP} = \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}$$

$$\bar{n} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{proj}_{\bar{n}} \overline{AP} = \frac{\vec{n} \cdot \overline{AP}}{\|\bar{n}\|^2} \vec{n}$$

$$= \frac{-10}{6} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$= \frac{-5}{3} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

(Vectorization of)
Desired point $= \vec{P} - \text{proj}_{\bar{n}} \vec{AP}$

$$= \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix} + \frac{5}{3} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 14/3 \\ -4/3 \\ -7/3 \end{bmatrix}$$

The point is $\left(\frac{14}{3}, \frac{-4}{3}, \frac{-7}{3} \right)$

④ $\bar{x} = \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix} + \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} s + \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix} t$

$\bar{n} = \bar{u} \times \bar{v}$, where $\bar{u}$ and $\bar{v}$ are direction vectors

$= \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \times \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}$

$$\boxed{\begin{matrix} 4 & -7 & 3 & 4 & -7 \\ 4 & 9 & 2 & 4 & 9 \end{matrix}}$$

$= \begin{bmatrix} -41 \\ 4 \\ 64 \end{bmatrix}$

Now $\bar{n} \cdot \bar{x} = \bar{n} \cdot \vec{p}$

$\begin{bmatrix} -41 \\ 4 \\ 64 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -41 \\ 4 \\ 64 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix}$

$-41x + 4y + 64z = 208$

⑤

$$\begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left[\begin{array}{ccc|c} 3 & -9 & 12 & 15 \\ -1 & 4 & 2 & c \\ 3 & 3 & 85 & -19 \end{array}\right] \end{array}$$

$\frac{R_1}{3}$ $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 5 \\ -1 & 4 & 2 & c \\ 3 & 3 & 85 & -19 \end{array}\right]$

$\begin{matrix} \\ R_2 + R_1 \\ R_3 - 3R_1 \end{matrix}$ $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 5 \\ 0 & 1 & 6 & 5+c \\ 0 & 12 & 73 & -34 \end{array}\right]$

$$\begin{array}{l} R_1+3R_2 \\ \\ R_3-12R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 22 & 20+3c \\ 0 & 1 & 6 & 5+c \\ 0 & 0 & 1 & -94-12c \end{array}\right]$$

$$\begin{array}{l} R_1-22R_3 \\ R_2-6R_3 \\ \\ \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2088+267c \\ 0 & 1 & 0 & 569+73c \\ 0 & 0 & 1 & -94-12c \end{array}\right] \begin{array}{l} \leftarrow 20+3c-22(-94-12c) \\ \leftarrow 5+c-6(-94-12c) \\ \\ \end{array}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2088+267c \\ 569+73c \\ -94-12c \end{bmatrix}$$