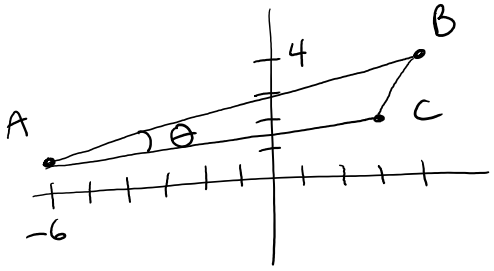
(1)



$$\bar{u} = \overrightarrow{AB} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

$$\bar{v} = \overrightarrow{AC} = \begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$\theta = \cos^{-1}\left(\frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|\,\|\bar{v}\|}\right)$$

$$= \cos^{-1}\left(\frac{93}{\sqrt{109}\sqrt{82}}\right)$$

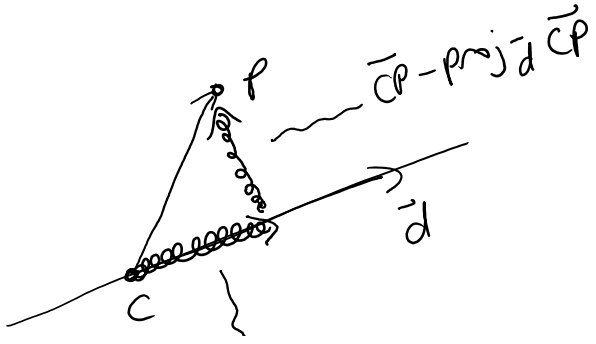
$$\simeq 10^\circ$$

(2) $V(\text{parallelepiped}) = 1\begin{vmatrix} 4 & -8 & 7 \\ 5 & c & 3 \\ 1 & -1 & 6 \end{vmatrix}1$

$$= |\,4(6c+3) + 8(27) + 7(-5-c)\,|$$

$$= |\,193 + 17c\,|$$

(3)



$$\overrightarrow{CP} = \begin{bmatrix} 7 \\ 7 \\ -10 \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$\text{proj}_{\vec{d}} \overrightarrow{CP} = \frac{\vec{d} \cdot \overrightarrow{CP}}{\|\vec{d}\|^2} \vec{d}$$

$$= \frac{41}{9} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$\overrightarrow{CP} - \text{proj}_{\vec{d}} \overrightarrow{CP} = \begin{bmatrix} 7 \\ 7 \\ -10 \end{bmatrix} - \frac{41}{9} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 22/9 \\ -19/9 \\ -8/9 \end{bmatrix}$$

$$= \frac{1}{9} \begin{bmatrix} 22 \\ -19 \\ -8 \end{bmatrix}$$

$$\text{distance} = \left\| \frac{1}{9} \begin{bmatrix} 22 \\ -19 \\ -8 \end{bmatrix} \right\|$$

$$= \frac{1}{9}\sqrt{909} \quad \text{or} \quad \frac{\sqrt{101}}{3}$$

(4) $$\vec{x} = \begin{bmatrix} -1 \\ 7 \\ 8 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix} s + \begin{bmatrix} 3 \\ 6 \\ 8 \end{bmatrix} t$$

$\vec{n} = \vec{u} \times \vec{v}$, where $\vec{u}$ and $\vec{v}$ are direction vectors

$$= \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix} \times \begin{bmatrix} 3 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{matrix} 2 & -3 & 8 & 2 & -3 \\ 3 & 6 & 8 & 3 & 6 \end{matrix}$$

$$= \begin{bmatrix} -72 \\ 8 \\ 21 \end{bmatrix}$$

Now $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix} -72 \\ 8 \\ 21 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -72 \\ 8 \\ 21 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 7 \\ 8 \end{bmatrix}$$

$$-72x + 8y + 21z = 296$$

(5) a)

$$\begin{matrix} & x & y & z & \\ & 2 & -6 & 8 & 44 \\ & -1 & 4 & 2 & 45 \\ & 3 & 3 & 85 & 872+c \end{matrix}$$

$\frac{R_1}{2}$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 22 \\ -1 & 4 & 2 & 45 \\ 3 & 3 & 85 & 872+c \end{array}\right]$$

R_2+R_1, R_3-3R_1

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 22 \\ 0 & 1 & 6 & 67 \\ 0 & 12 & 73 & 806+c \end{array}\right]$$

R_1+3R_2

$$\left[\begin{array}{ccc|c} 1 & 0 & 22 & 223 \\ 0 & 1 & 6 & 67 \\ 0 & 0 & 1 & 2+c \end{array}\right]$$

R_3-12R_2

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1179-77c \end{array}\right] \leftarrow 223-22(2+c)$$

$R_3 - 12R_2$

$R_1 - 22R_3$
$R_2 - 6R_3$

$$\begin{bmatrix} 1 & 0 & 0 & | & 179-22c \\ 0 & 1 & 0 & | & 55-6c \\ 0 & 0 & 1 & | & 2+c \end{bmatrix} \leftarrow 223-22(2+c)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 179-22c \\ 55-6c \\ 2+c \end{bmatrix}$$

b)

$y = -56$

$55 - 6c = -56$

$111 = 6c$

$c = 18.5 \rightarrow x = 179 - 22c$

$x = -228$