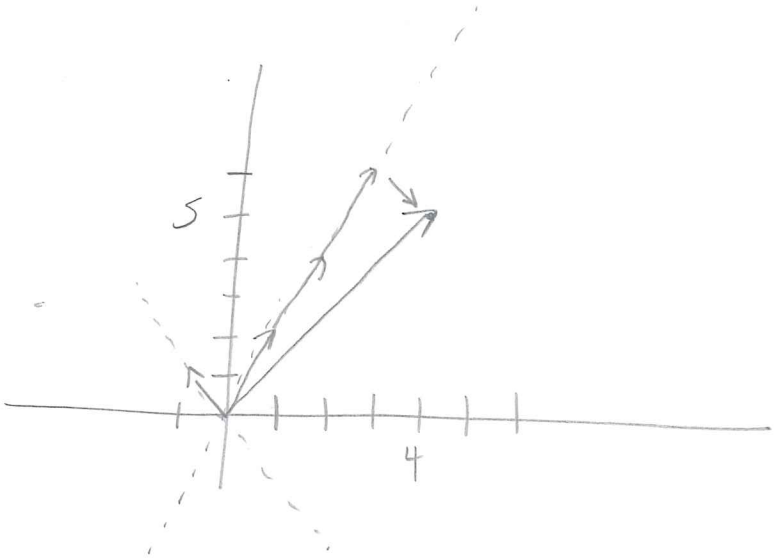
1. [2 marks] Let $[4,5] = c_1[1,2] + c_2[-1,1]$.
Find the coefficients c_1 and c_2 by graphing. Show all your work.



$[4,5] = 3[1,2] - [-1,1]$

2. [5 marks] Let $\mathbf{u} = [9, 2]$ and $\mathbf{v} = [-4, 7]$ Find:

a) the angle between $\mathbf{u}$ and $\mathbf{v}$ [3]

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$-22 = \sqrt{85}\sqrt{65}\cos\theta$$

$$\cos\theta = \frac{-22}{\sqrt{85}\sqrt{65}}$$

$$\theta = \cos^{-1}\left(\frac{-22}{\sqrt{85}\sqrt{65}}\right)$$

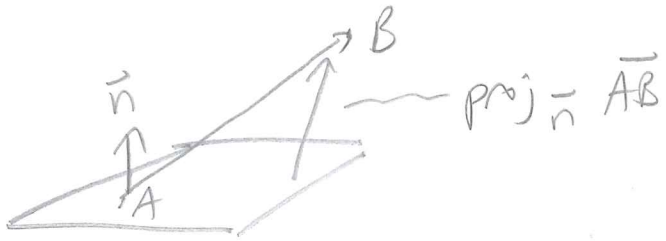
$$\approx 107^\circ$$

b) the projection of $\mathbf{v}$ onto $\mathbf{u}$ [2]

$$\text{proj}_{\vec{u}}\vec{v} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2}\vec{u}$$

$$= \frac{-22}{85}[9, 2]$$

3. [4 marks] Find the distance between the plane $2x - 5y + z = 13$ and the point $B = (2, -1, 6)$



A: any point on plane

$A = (0, 0, 13)$

$\overrightarrow{AB} = [2, -1, -7]$ (1)

$\vec{n} = [2, -5, 1]$ (1)

$$\text{proj}_{\vec{n}} \overrightarrow{AB} = \frac{\vec{n} \cdot \overrightarrow{AB}}{\|\vec{n}\|^2} \vec{n}$$

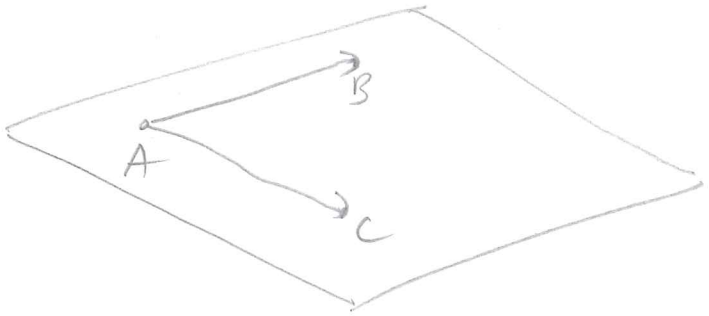
$$= \frac{2}{30} [2, -5, 1] \quad (1)$$

$$\text{distance} = \|\text{proj}_{\vec{n}} \overrightarrow{AB}\|$$

$$= \frac{2\sqrt{30}}{30} \text{ or } \frac{\sqrt{30}}{15} \quad (1)$$

≈ 0.37

4. [4 marks] Find the general form of the plane through points $A = (3, -6, 7)$, $B = (3, 0, 9)$ and $C = (-5, 1, 1)$



(1)

$$\vec{AB} = [0, 6, 2]$$
$$\vec{AC} = [-8, 7, -6]$$

(1)

$$\vec{n} = \vec{AB} \times \vec{AC}$$
$$= [-50, -16, 48]$$

$$\begin{matrix} 0 & 6 & 2 & 0 & 6 \\ -8 & 7 & -6 & -8 & 7 \end{matrix}$$

(2)

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{P}$$

$\vec{P}$: any point on plane

$$\begin{bmatrix} -50 \\ -16 \\ 48 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -50 \\ -16 \\ 48 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -6 \\ 7 \end{bmatrix}$$

$$-50x - 16y + 48z = 282$$

5. [4 marks] Solve using Gauss-Jordan Elimination:

$$\begin{aligned} 2x - 8y - 68z &= -70 \\ 2x - 7y - 61z &= -57 \\ -6x + 27y + 225z &= 249 \end{aligned}$$

$$\begin{matrix} x & y & z \end{matrix}$$

$$\left[\begin{array}{ccc|c} 2 & -8 & -68 & -70 \\ 2 & -7 & -61 & -57 \\ -6 & 27 & 225 & 249 \end{array}\right]$$

$R_1/2$

$$\left[\begin{array}{ccc|c} 1 & -4 & -34 & -35 \\ 2 & -7 & -61 & -57 \\ -6 & 27 & 225 & 249 \end{array}\right]$$

$R_2 - 2R_1$
$R_3 + 6R_1$

$$\left[\begin{array}{ccc|c} 1 & -4 & -34 & -35 \\ 0 & 1 & 7 & 13 \\ 0 & 3 & 21 & 39 \end{array}\right]$$

$R_1 + 4R_2$

$R_3 - 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -6 & 17 \\ 0 & 1 & 7 & 13 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\uparrow$

$z = t$

$x - 6z = 17 \rightarrow x = 17 + 6t$

$y + 7z = 13 \rightarrow y = 13 - 7t$

$$\vec{x} = \begin{bmatrix} 17 \\ 13 \\ 0 \end{bmatrix} + t \begin{bmatrix} 6 \\ -7 \\ 1 \end{bmatrix}$$