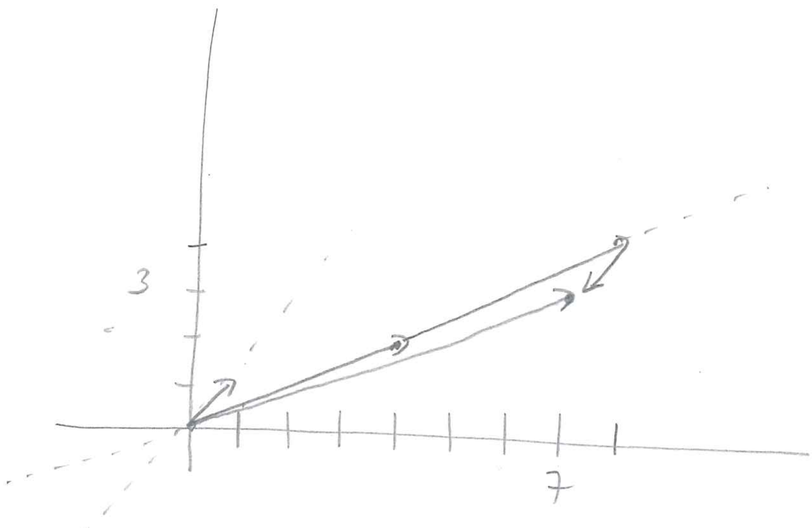
1. [2 marks] Let $[7,3] = c_1[4,2] + c_2[1,1]$.
Find the coefficients c_1 and c_2 by graphing. Show all your work.



$[7,3] = 2[4,2] - [1,1]$

2. [5 marks] Let $\mathbf{u} = [7, 8]$ and $\mathbf{v} = [-6, 3]$ Find:

a) the angle between $\mathbf{u}$ and $\mathbf{v}$

[3]

$$\bar{u} \cdot \bar{v} = \|\bar{u}\| \|\bar{v}\| \cos\theta$$

$$-18 = \sqrt{113}\sqrt{45} \cos\theta$$

$$\frac{-18}{\sqrt{113}\sqrt{45}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-18}{\sqrt{113}\sqrt{45}}\right)$$

$$\simeq 105^\circ$$

b) the projection of $\mathbf{v}$ onto $\mathbf{u}$

[2]

$$\text{proj}_{\bar{u}} \bar{v} = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|^2} \bar{u}$$

$$= \frac{-18}{113} [7, 8]$$

3. [4 marks] Find the distance between the plane $3x - 5y + z = 17$ and the point $B = (1, -6, 2)$

A = any point on plane

$A = (0, 0, 17)$

$\vec{AB} = [1, -6, -15]$ (1)

$\vec{n} = [3, -5, 1]$ (1)

$$\text{proj}_{\vec{n}} \vec{AB} = \frac{\vec{n} \cdot \vec{AB}}{\|\vec{n}\|^2} \vec{n}$$

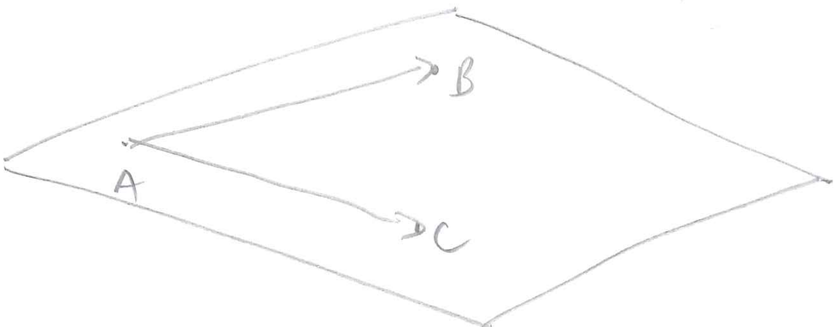
$$= \frac{18}{35} [3, -5, 1]$$ (1)

$$\text{distance} = \| \text{proj}_{\vec{n}} \vec{AB} \|$$

$$= \frac{18\sqrt{35}}{35}$$ (1)

≈ 3.04

4. [4 marks] Find the general form of the plane through points $A = (1, -2, 4)$, $B = (3, 0, 9)$ and $C = (-5, 1, 1)$



(1) $\overline{AB} = [2, 2, 5]$

$\overline{AC} = [-6, 3, -3]$

(1) $\overline{n} = \overline{AB} \times \overline{AC}$

$= [-21, -24, 18]$

$\overline{n} \cdot \overline{x} = \overline{n} \cdot \overline{P}$ ← P: any point on plane

(2)

$$\begin{bmatrix} -21 \\ -24 \\ 18 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -21 \\ -24 \\ 18 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$-21x - 24y + 18z = 99$

5. [4 marks] Solve using Gauss-Jordan Elimination:

$$\begin{aligned} 2x - \ 8y - \ 68z &= -58 \\ 2x - \ 7y - \ 61z &= -46 \\ -6x + 27y + 225z &= 210 \end{aligned}$$

$$\left[\begin{array}{ccc|c} x & y & z & \\ 2 & -8 & -68 & -58 \\ 2 & -7 & -61 & -46 \\ -6 & 27 & 225 & 210 \end{array}\right]$$

$R_1/2$

$$\left[\begin{array}{ccc|c} 1 & -4 & -34 & -29 \\ 2 & -7 & -61 & -46 \\ -6 & 27 & 225 & 210 \end{array}\right]$$

R_2-2R_1
R_3+6R_1

$$\left[\begin{array}{ccc|c} 1 & -4 & -34 & -29 \\ 0 & 1 & 7 & 12 \\ 0 & 3 & 21 & 36 \end{array}\right]$$

R_1+4R_2
R_3-3R_2

$$\left[\begin{array}{ccc|c} 1 & 0 & -6 & 19 \\ 0 & 1 & 7 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

↑
$z=t$

$x-6z=19 \rightarrow x=19+6t$
$y+7z=12 \rightarrow y=12-7t$

$$\bar{x} = \begin{bmatrix} 19 \\ 12 \\ 0 \end{bmatrix} + t\begin{bmatrix} 6 \\ -7 \\ 1 \end{bmatrix}$$