(14) Write $\vec{v} = c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3}$

$$\begin{array}{ccc} c_1 & c_2 & c_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 13 \\ 7 & 2 & 2 & -6 \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 7 & 2 & 2 & -6 \\ 0 & 0 & 1 & 13 \end{array}\right]$$

$$R_2 - 7R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 2 & 2 & -90 \\ 0 & 0 & 1 & 13 \end{array}\right]$$

$$R_2/2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 1 & -45 \\ 0 & 0 & 1 & 13 \end{array}\right]$$

$$\vec{v} = 12\vec{v_1} - 58\vec{v_2} + 13\vec{v_3}$$

$$R_2 - R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & -58 \\ 0 & 0 & 1 & 13 \end{array}\right]$$

$$T(\vec{v}) = T\left(12\vec{v_1} - 58\vec{v_2} + 13\vec{v_3}\right)$$
$$= 12T(\vec{v_1}) - 58T(\vec{v_2}) + 13T(\vec{v_3})$$
because T is linear
$$= 12\begin{bmatrix} -6 \\ 30 \\ 24 \\ 8 \end{bmatrix} - 58\begin{bmatrix} -2 \\ 8 \\ 6 \\ 2 \end{bmatrix} + 13\begin{bmatrix} -1 \\ 9 \\ 6 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 31 \\ 13 \\ 18 \\ -20 \end{bmatrix}$$

(15)

$$|A| = \begin{vmatrix} 2 & 1 & 6 & 9 \\ 2 & 1 & 3 & 0 \\ 6 & 8 & 1 & 0 \\ 4 & 12 & 1 & 1 \end{vmatrix}$$

Get an upper triangular determinant.

$R_2 - R_1$, $R_3 - 3R_1$, $R_4 - 2R_1$

$$= \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 0 & -3 & -9 \\ 0 & 5 & -17 & -27 \\ 0 & 10 & -11 & -17 \end{vmatrix}$$

$R_2 \leftrightarrow R_3$

$$= - \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & -3 & -9 \\ 0 & 10 & -11 & -17 \end{vmatrix}$$

$R_4 - 2R_2$

$$= - \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 23 & 37 \end{vmatrix}$$

$R_3 / (-3)$

$$= 3 \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 23 & 37 \end{vmatrix}$$

$R_4 - 23R_3$

$$= 3 \begin{vmatrix} 2 & 1 & 6 & 9 \\ 0 & 5 & -17 & -27 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -32 \end{vmatrix}$$

upper triangular ✓

So $|A| = 3(2 \cdot 5 \cdot 1 \cdot (-32))$
$= -960$

(16)

$$\text{Matrix of Cofactors} = \begin{bmatrix} 0 & 2 & -1 \\ -21 & 5 & 1 \\ 7 & -2 & 1 \end{bmatrix}$$ & transpose

$$\text{adj}(A) = \begin{bmatrix} 0 & -21 & 7 \\ 2 & 5 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$= \frac{1}{7} \begin{bmatrix} 0 & -21 & 7 \\ 2 & 5 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

expanding along col. 1

$$|A| = 1 \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} + 1 \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1(0) + 1(7)$$

$$= 7$$

(17) a) $$|A-\lambda I| = \begin{vmatrix} 2-\lambda & 1 & 1 & 6 \\ 0 & 4-\lambda & 2 & 0 \\ 0 & 0 & 2-\lambda & 8 \\ 0 & 0 & 0 & 2-\lambda \end{vmatrix}$$

$$= (2-\lambda)^3(4-\lambda)$$

or $(\lambda-2)^3(\lambda-4)$

b) $[A-2I \mid \vec{0}]$

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 6 & 0 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 - 2R_1$ $$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 6 & 0 \\ 0 & 0 & 0 & -12 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2/(-12)$ $$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 6 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 - 6R_2$

$R_3 - 8R_2$ $$\left[\begin{array}{cccc|c} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{x_1 = s}$ $\boxed{x_3 = t}$

$x_2 + x_3 = 0 \rightarrow \boxed{x_2 = -t}$

$\boxed{x_4 = 0}$

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} s + \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} t \quad \text{Basis} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

c) The algebraic multiplicity of $\lambda=2$ is 3 [from $(\lambda-2)^3$]

but the geometric multiplicity of $\lambda=2$ is only 2 [E_2 has 2 basis vectors].

Need alg. mult = geo. mult for each eigenvalue in order for A to be diagonalizable.

(18) A is upper triangular

$\Rightarrow \lambda = 1, 0, -3$

E_1 : $[A - I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 0 & 0 & 2 & 0 \\ 0 & -1 & 6 & 0 \\ 0 & 0 & -4 & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = t$

$x_2 = 0$

$x_3 = 0$

$\bar{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t$ Basis $= \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

E_0 : $[A - 0I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_2 = t$

$x_1 = x_3 = 0$

$\bar{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} t$ Basis $= \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

E_{-3} : $[A+3I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 4 & 0 & 2 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1/4$
$R_2/3$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\uparrow$

$\boxed{x_3 = t}$

$x_2 + 2x_3 = 0$

$\boxed{x_2 = -2t}$

$x_1 + \frac{1}{2} x_3 = 0$

$\boxed{x_1 = -\frac{t}{2}}$

$\vec{x} = \begin{bmatrix} -\frac{1}{2} \\ -2 \\ 1 \end{bmatrix} t$

or $\vec{x} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} t$

$E_{-3} = \text{span}\left(\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}\right)$

$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -2 \end{bmatrix}$

or $P = \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

or $P = \begin{bmatrix} 0 & 1 & -\frac{1}{2} \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ etc.

(19) Since $\mathcal{B}$ is an orthogonal basis
$\{\vec{b_1}, \vec{b_2}, \vec{b_3}\}$

$$\vec{u} = \frac{\vec{u}\cdot\vec{b_1}}{\|\vec{b_1}\|^2}\vec{b_1} + \frac{\vec{u}\cdot\vec{b_2}}{\|\vec{b_2}\|^2}\vec{b_2} + \frac{\vec{u}\cdot\vec{b_3}}{\|\vec{b_3}\|^2}\vec{b_3}$$

$$= \frac{-150}{75}\begin{bmatrix}1\\-5\\7\end{bmatrix} + \frac{342}{114}\begin{bmatrix}7\\7\\4\end{bmatrix} + \frac{2850}{8550}\begin{bmatrix}-69\\45\\42\end{bmatrix}$$

$$\left\|\begin{bmatrix}1\\-5\\7\end{bmatrix}\right\|^2 = 75$$

$$= -2\begin{bmatrix}1\\-5\\7\end{bmatrix} + 3\begin{bmatrix}7\\7\\4\end{bmatrix} + \frac{1}{3}\begin{bmatrix}-69\\45\\42\end{bmatrix}$$

$$= \begin{bmatrix}-4\\46\\12\end{bmatrix}$$

(20.) $W = \left\{ \begin{bmatrix} 5t \\ -2t \\ 10t \end{bmatrix} \right\}$

$= \text{span}\left(\begin{bmatrix} 5 \\ -2 \\ 10 \end{bmatrix} \right)$

Basis for $W^\perp$ = Basis for null(A)

where $A = \begin{bmatrix} 5 & -2 & 10 \end{bmatrix}$

Solve $\left[\begin{array}{ccc|c} 5 & -2 & 10 & 0 \end{array}\right]$

$R_1/5$ $\left[\begin{array}{ccc|c} 1 & \frac{-2}{5} & 2 & 0 \end{array}\right]$

$x_2 = s$ $x_3 = t$

$x_1 - \frac{2}{5}x_2 + 2x_3 = 0$

$x_1 = \frac{2}{5}s - 2t$

$\vec{x} = \begin{bmatrix} \frac{2}{5} \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} t$

Basis for $W^\perp = \left\{ \begin{bmatrix} \frac{2}{5} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

(21) a) $\vec{v_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad X = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$$\vec{v_2} = \begin{bmatrix} 1 \\ 5 \\ 6 \\ 0 \end{bmatrix} - \text{proj}_X \begin{bmatrix} 1 \\ 5 \\ 6 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 5 \\ 6 \\ 0 \end{bmatrix} - \frac{6}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix} \qquad X = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix} \right\}$$

$$\vec{v_3} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - \text{proj}_X \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - \frac{2}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{12}{44} \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix}$$

Can scale to simplify arithmetic:

$$11\vec{v_3} = 11 \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix} - 11 \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} -2 \\ 2 \\ 6 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 4 \\ 33 \end{bmatrix}$$

$\{\vec{v_1}, \vec{v_2}, 11\vec{v_3}\}$ is an <u>orthogonal</u> basis.

<u>Orthonormal</u> basis:

$$= \left\{ \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{11} \\ 1/\sqrt{11} \\ 3/\sqrt{11} \\ 0 \end{bmatrix}, \begin{bmatrix} 6/\sqrt{1177} \\ -6/\sqrt{1177} \\ 4/\sqrt{1177} \\ 33/\sqrt{1177} \end{bmatrix} \right\}$$

b) $Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{11} & 6/\sqrt{1177} \\ 1/\sqrt{2} & 1/\sqrt{11} & -6/\sqrt{1177} \\ 0 & 3/\sqrt{11} & 4/\sqrt{1177} \\ 0 & 0 & 33/\sqrt{1177} \end{bmatrix}$

and if $A = QR$

$Q^TA = Q^TQR \qquad [Q^TQ = I]$

$Q^TA = R$

$$R = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{11} & 1/\sqrt{11} & 3/\sqrt{11} & 0 \\ 6/\sqrt{1177} & -6/\sqrt{1177} & 4/\sqrt{1177} & 33/\sqrt{1177} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2/\sqrt{2} & 6/\sqrt{2} & 2/\sqrt{2} \\ 0 & 22/\sqrt{11} & 6/\sqrt{11} \\ 0 & 0 & 107/\sqrt{1177} \end{bmatrix}$$

Check: $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{11}} & \frac{6}{\sqrt{1177}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{11}} & \frac{-6}{\sqrt{1177}} \\ 0 & \frac{3}{\sqrt{11}} & \frac{4}{\sqrt{1177}} \\ 0 & 0 & \frac{33}{\sqrt{1177}} \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt{2}} & \frac{6}{\sqrt{2}} & \frac{2}{\sqrt{2}} \\ 0 & \frac{22}{\sqrt{11}} & \frac{6}{\sqrt{11}} \\ 0 & 0 & \frac{107}{\sqrt{1177}} \end{bmatrix}$ ✓

(22) Eigenvalues of A are $\lambda = 2, 5$

E_2: $[A - 2I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_2$ $$\left[\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array}\right]$$

$R_1/(-1)$ $$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - 2R_1 \\ R_3 + R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right]$$

$R_2/3$ $$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 + 2R_2 \\ \\ R_3 + 3R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\boxed{x_3 = t}$

$x_1 - x_3 = 0$ $\boxed{x_1 = t}$

$x_2 - x_3 = 0$ $\boxed{x_2 = t}$

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

Orthonormal basis for E_2: $$\left\{ \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \right\}$$

E_5: $[A-5I|\vec{0}]$

$$\left[\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ -1 & -1 & -1 & 0 \\ -1 & -1 & -1 & 0 \end{array}\right]$$

$R_1/(-1)$
R_2-R_1
R_3-R_1

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_2=s$ $x_3=t$

$x_1+x_2+x_3=0$ $\boxed{x_1=-s-t}$

$$\vec{x}=\begin{bmatrix}-1\\1\\0\end{bmatrix}s+\begin{bmatrix}-1\\0\\1\end{bmatrix}t$$

Basis for $E_5 = \left\{\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\0\\1\end{bmatrix}\right\}$

orthonormal basis:

$\vec{v_1}=\begin{bmatrix}-1\\1\\0\end{bmatrix}$ $X=\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$

$\vec{v_2}=\begin{bmatrix}-1\\0\\1\end{bmatrix}-\text{proj}_X\begin{bmatrix}-1\\0\\1\end{bmatrix}$

$=\begin{bmatrix}-1\\0\\1\end{bmatrix}-\frac{1}{2}\begin{bmatrix}-1\\1\\0\end{bmatrix}$

$2\vec{v_2}=2\begin{bmatrix}-1\\0\\1\end{bmatrix}-\begin{bmatrix}-1\\1\\0\end{bmatrix}$

$=\begin{bmatrix}-1\\-1\\2\end{bmatrix}$

$\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\-1\\2\end{bmatrix}\right\}$

Orthonormal basis:

$\left\{\begin{bmatrix}\frac{-1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\\0\end{bmatrix},\begin{bmatrix}\frac{-1}{\sqrt{6}}\\\frac{-1}{\sqrt{6}}\\\frac{2}{\sqrt{6}}\end{bmatrix}\right\}$

$$Q=\begin{bmatrix}\frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}}\end{bmatrix}$$

(23)

$$z = \frac{x+yi}{13+5i}$$

$$= \frac{(x+yi)}{(13+5i)} \cdot \frac{(13-5i)}{(13-5i)}$$

$$= \frac{13x+5y+(13y-5x)i}{169+25}$$

$$= \frac{13x+5y}{194} + \frac{13y-5x}{194}\, i$$

(24) $z = -8i$ $\quad |z|$ must be ≥ 0

$= 8[0 - i]$

$$\left.\begin{array}{l}\cos\theta = 0 \\ \sin\theta = -1\end{array}\right\} \theta = \frac{3\pi}{2}$$

$$= 8\left[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right]$$

$$z^{1/3} = 8^{1/3}\left[\cos\frac{\frac{3\pi}{2} + 2\pi\alpha}{3} + i\sin\frac{\frac{3\pi}{2} + 2\pi\alpha}{3}\right]$$

$\alpha = 0, 1, 2$

α	$\frac{3\pi + 4\pi\alpha}{6}$
0	$\frac{\pi}{2}$
1	$\frac{7\pi}{6}$
2	$\frac{11\pi}{6}$

$$z_1^{1/3} = 2\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right] = 2i$$

$$z_2^{1/3} = 2\left[\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right] = 2\left[-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right] = -\sqrt{3} - i$$

$$z_3^{1/3} = 2\left[\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right] = 2\left[\frac{\sqrt{3}}{2} - \frac{1}{2}i\right] = \sqrt{3} - i$$