

Math 251 X01 & X02 Assignment

Total: 14 marks

Name: _____

Due: At the beginning of class, Friday April 8th

Covers Sections: 4.3-4.4 and 5.1-5.4

INSTRUCTIONS

- * Print this paper and submit your work on this paper.
- * If you are away on the due date then submit via the D2L Dropbox.
- * You may discuss with others but your write-up must be your own work.
- * Show all your work for full marks.

1. [2 marks] Find all the eigenvalues of $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & -9 \\ 0 & 1 & -5 \end{bmatrix}$

2. [3 marks] The matrix A has eigenvalue -1 corresponding to the eigenvector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and eigenvalue 3 corresponding to the eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the top-left entry of A^n .

3. [3 marks] Let $W = \text{span}\left(\begin{bmatrix} 1 \\ -4 \\ 6 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 5 \\ 7 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -12 \\ 3 \\ -4 \end{bmatrix}\right)$. Find a basis for W^\perp .

4. [4 marks] Find an orthogonal basis for $\text{span}\left(\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}\right)$.

5. [2 marks] Use the spectral decomposition to find a matrix A with the following eigenvalues and eigenvectors:

$$\lambda_1 = 2 \text{ corresponding to } \mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 = 2 \text{ corresponding to } \mathbf{x}_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\lambda_3 = 4 \text{ corresponding to } \mathbf{x}_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$