(1) Let x = # Weeks Mine A must operate
y = " B "
z = " C "

$$\begin{array}{cccc|cc} x & y & z & & & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 12 & 5 & 7 & 69 \\ 20 & 4 & 9 & 100 \\ 3 & 2 & 8 & 43 \end{array}\right] \begin{array}{l} \text{(anthracite)} \\ \text{(ordinary)} \\ \text{(bituminous)} \end{array}$$

(2) Let $c_1\bar{u}_1 + c_2\bar{u}_2 + c_3\bar{u}_3 = \bar{0}$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 4 & -1 & 0 \end{array}\right]$$

$$R_2/3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 4 & -1 & 0 \end{array}\right]$$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \end{array}\right.$$

$$\begin{array}{cc} & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ R_3-4R_2 & \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \end{array}\right] \end{array} \quad \text{REF}$$

The only solution is $c_1 = c_2 = c_3 = 0$

The vectors are linearly independent.

(3) Let $c_1\begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix} + c_2\begin{bmatrix} 2 & 8 \\ 7 & 14 \end{bmatrix} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$

Each zero row of the REF will produce a condition on w, x, y, z.

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & 2 & w \\ 4 & 8 & x \\ 3 & 7 & y \\ 6 & 14 & z \end{array}\right] \end{array}$$

$$\begin{array}{c} \\ R_2-4R_1 \\ R_3-3R_1 \\ R_4-6R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 0 & x-4w \\ 0 & 1 & y-3w \\ 0 & 2 & z-6w \end{array}\right]$$

$$\begin{array}{c} \\ \\ \\ R_4-2R_3 \end{array} \left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 0 & x-4w \\ 0 & 1 & y-3w \\ 0 & 0 & z-2y \end{array}\right]$$

$(z-6w)-2(y-3w)$

Reorder rows
$$\left[\begin{array}{cc|c} 1 & 2 & w \\ 0 & 1 & y-3w \\ 0 & 0 & x-4w \\ 0 & 0 & z-2y \end{array}\right] \quad \text{REF}$$

Consistent system $\Rightarrow$ $x-4w=0 \rightarrow x=4w$

AND $z-2y=0 \rightarrow z=2y$

$$\text{span}\left(\begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 2 & 8 \\ 7 & 14 \end{bmatrix}\right)$$

$$= \left\{ \begin{bmatrix} w & x \\ y & z \end{bmatrix} \text{ such that } x=4w,\ z=2y \right\}$$

$$= \left\{ \begin{bmatrix} w & 4w \\ y & 2y \end{bmatrix} \right\}$$