Math 251 X02
Test Three

Time: 50 minutes
Total: 17 marks

Name: _____

1. [3 marks] Solve:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 0 & 4 & -3 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 38 \\ 63 \\ 96 \end{bmatrix}$$

2. [3 marks] Use Cramer's Rule to find y:

$$\begin{aligned} -y + z &= -5 \\ 4x + 3y + 7z &= 33 \\ 2x + y - 8z &= 2 \end{aligned}$$

3. [3 marks] Find all eigenvectors of $A = \begin{bmatrix} 6 & 5 \\ -8 & -8 \end{bmatrix}$ corresponding to $\lambda = 2$

4. [3 marks] The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ first rotates a vector by 120° counter-clockwise then performs the transformation S below. Find the standard matrix for T.

$$S\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x - 8y \\ 6x \end{bmatrix}$$

5. [3 marks] $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 9 & 1 \end{bmatrix}$ has RREF$= \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

Find a basis for:

a) the column space of A

b) the row space of A

c) the null space of A

6. [2 marks] A is a 5×8 matrix.

a) What are the possible values of rank(A)?

b) What are the possible values of nullity(A)?