1. [3 marks]

Write $\mathbf{w} = \begin{bmatrix} 13 \\ 76 \\ 74 \end{bmatrix}$ as a linear combination of $\mathbf{u} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 11 \\ 10 \end{bmatrix}$, or show that it is impossible to do so.

$$c_1 \vec{u} + c_2 \vec{v} = \vec{w}$$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & 13 \\ 5 & 11 & 76 \\ 4 & 10 & 74 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - 5R_1 \\ R_3 - 4R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 13 \\ 0 & 1 & 11 \\ 0 & 2 & 22 \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 - 2R_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & -9 \\ 0 & 1 & 11 \\ 0 & 0 & 0 \end{array}\right]$$

$$c_1 = -9, \quad c_2 = 11$$

$$-9\vec{u} + 11\vec{v} = \vec{w}$$

2. [3 marks] Write down the system of equations you would use to solve the following problem. **Do not solve the system.**

Balance $C_{57}H_{110}O_6 + O_2 \to CO_2 + H_2O$

$$w(C_{57}H_{110}O_6) + xO_2 \to yCO_2 + zH_2O$$

C: $57w = y$

H: $110w = 2z$

O: $6w + 2x = 2y + z$

OR

$$\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \left[\begin{array}{cccc|c} 57 & 0 & -1 & 0 & 0 \\ 110 & 0 & 0 & -2 & 0 \\ 6 & 2 & -2 & -1 & 0 \end{array}\right] \end{array}$$

3. [4 marks] Compute $B^2 - AC^T$ where:

$A = \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & -3 \\ 9 & 2 \end{bmatrix}$

$$B^2 - AC^T = \begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 5 & 9 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 78 & 42 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 8 & 7 \\ -3 & 39 \end{bmatrix}$$

$$= \begin{bmatrix} 70 & 35 \\ 15 & -21 \end{bmatrix}$$

4. [3 marks] Write $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ as a product of elementary matrices.

$\frac{R_2}{2}$ $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$R_1 - 3R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$$\underbrace{E_2 E_1}_{A^{-1}} A = I$$

$$A^{-1} = E_2 E_1$$

$$A = (E_2 E_1)^{-1}$$

$$= E_1^{-1} E_2^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

5. [3 marks] Find a $2 \times 2$ matrix $A$ such that:

a) $A^2 = I$ but $A \neq I$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

b) $A \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 7 & -2 \end{bmatrix}$$