

1. [3 marks]

Write  $\mathbf{w} = \begin{bmatrix} 13 \\ 76 \\ 74 \end{bmatrix}$  as a linear combination of  $\mathbf{u} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$  and  $\mathbf{v} = \begin{bmatrix} 2 \\ 11 \\ 10 \end{bmatrix}$ ,  
or show that it is impossible to do so.

$$c_1 \vec{u} + c_2 \vec{v} = \vec{w}$$

$$\begin{bmatrix} c_1 & c_2 & | & 13 \\ 1 & 2 & | & 76 \\ 5 & 11 & | & 74 \\ 4 & 10 & | & 74 \end{bmatrix}$$

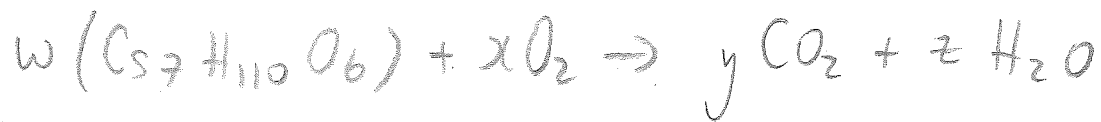
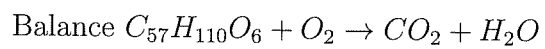
$$\begin{array}{l} R_2 - 5R_1 \\ R_3 - 4R_1 \end{array} \begin{bmatrix} 1 & 2 & | & 13 \\ 0 & 1 & | & 11 \\ 0 & 2 & | & 22 \end{bmatrix}$$

$$\begin{array}{l} R_1 - 2R_2 \\ R_3 - 2R_2 \end{array} \begin{bmatrix} 1 & 0 & | & -9 \\ 0 & 1 & | & 11 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$$c_1 = -9, \quad c_2 = 11$$

$$-9\vec{u} + 11\vec{v} = \vec{w}$$

2. [3 marks] Write down the system of equations you would use to solve the following problem. Do not solve the system.



$$C: 57w = y$$

$$H: 110w = 2z$$

$$O: 6w + 2x = 2y + z$$

OR

$$\begin{array}{cccc|c} w & x & y & z & \\ \hline 57 & 0 & -1 & 0 & 0 \\ 110 & 0 & 0 & -2 & 0 \\ 6 & 2 & -2 & -1 & 0 \end{array}$$

3. [4 marks] Compute  $B^2 - AC^T$  where:

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}, B = \begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 5 & -3 \\ 9 & 2 \end{bmatrix}$$

$$B^2 - AC^T = \begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 5 & 9 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 78 & 42 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 8 & 7 \\ -3 & 39 \end{bmatrix}$$

$$= \begin{bmatrix} 70 & 35 \\ 15 & -21 \end{bmatrix}$$

4. [3 marks] Write  $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$  as a product of elementary matrices.

$$\frac{R_2}{2} \quad \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \quad E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \quad E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$R_1 - 3R_2 \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} \quad E_2^{-1} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\underbrace{E_2 E_1}_A A = I$$

$A^{-1}$

$$A^{-1} = E_2 E_1$$

$$A = (E_2 E_1)^{-1}$$

$$= E_1^{-1} E_2^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

5. [3 marks] Find a  $2 \times 2$  matrix  $A$  such that:

a)  $A^2 = I$  but  $A \neq I$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

b)  $A \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 7 & -2 \end{bmatrix}$$