Math 251 X02

Test Three

Time: 50 minutes

Total: 20 marks

Name: ______________________________

1. [3 marks] Solve:

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 6 & 1 & 3 \\ 0 & 2 & -2 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \\ 47 \end{bmatrix}$$

2. [3 marks] A and B are 4×4 matrices. Given that $\det(A) = a$ and $\det(B) = b$, find $\det(2AB^T)$.

3. [3 marks] Find all the eigenvalues of $A = \begin{bmatrix} 7 & -2 \\ -1 & 6 \end{bmatrix}$.

4. [4 marks] Find all eigenvectors of $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ corresponding to $\lambda = -2$.

5. [3 marks] $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 10 & 10 \\ 4 & 8 & 13 & 18 \end{bmatrix}$ has RREF$= \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Find:

a) a basis for the column space of A

b) $\dim(\text{null}(A))$

6. [4 marks] Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 8 \\ 17 \end{bmatrix}$.

T is a linear transformation such that $T(\mathbf{v}_1) = \begin{bmatrix} 2 \\ -9 \end{bmatrix}$ and $T(\mathbf{v}_2) = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$.

Find $T(\begin{bmatrix} 109 \\ 233 \end{bmatrix})$.